



HAL
open science

Computing natural recharge using the water-table fluctuation method: where to site an observation well

Jean-Christophe Maréchal, Pierre Perrochet, Yvan Caballero

► To cite this version:

Jean-Christophe Maréchal, Pierre Perrochet, Yvan Caballero. Computing natural recharge using the water-table fluctuation method: where to site an observation well. *Hydrogeology Journal*, 2023, 10.1007/s10040-023-02707-5 . hal-04228361

HAL Id: hal-04228361

<https://brgm.hal.science/hal-04228361>

Submitted on 4 Oct 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1

2 **Computing natural recharge using the water-table fluctuation method: where**
3 **to site an observation well**

4

5 Jean-Christophe Maréchal ^{1,2*}, Pierre Perrochet ³, Yvan Caballero ^{1,2}

6 ¹ BRGM, Univ Montpellier, Montpellier, France

7 ² G-eau, UMR 183, INRAE, CIRAD, IRD, AgroParisTech, Institut Agro, BRGM, Montpellier, France

8 ³ CHYN, Univ Neuchâtel, Neuchâtel, Suisse

9 * corresponding author, jc.marechal@brgm.fr

10 ORCID id : Maréchal (0000-0001-8179-1294), Caballero (0000-0002-5328-7338)

11 **Abstract**

12 Accurate evaluation of natural recharge is fundamental for the estimation of renewable
13 groundwater resources and constitutes a key issue for water resources management. Among
14 physical, chemical and numerical methods, the water-table fluctuation technique is simple
15 and easy to use. The main question is where, with respect to the draining stream, to locate
16 the monitoring well to effectively observe representative water level fluctuations. Storage and
17 discharged volumes in an unconfined aquifer during the water-table fluctuation application
18 are described with equations using the Dupuit-Forchheimer model. This leads to
19 determination of an optimal distance from the stream (0.4 times the length of the aquifer) for
20 siting observation wells. Beyond this distance, the computed volumes stored in the aquifer

21 are overestimated by up to 20%. Less than this distance, the stored volume is highly
22 underestimated.

23 **Keywords:** water table fluctuation, piezometry, recharge, discharge, analytical solution

24

25 **1. Introduction**

26 The estimation of natural recharge, defined as the precipitation infiltration to the water table, is a
27 fundamental issue in hydrogeology. It provides a first step towards the estimation of renewable
28 groundwater resources and constitutes a key issue for overall water resources management. Several
29 methods are used to estimate the natural recharge: physical, chemical or numerical. The reliability of
30 recharge estimates using different techniques is variable (Scanlon et al. 2002; Scanlon 2011). Among
31 these methods, the water-table fluctuation (WTF) technique relies on the measurement of
32 groundwater storage and associated water level changes induced by recharge during a given period
33 (Healy and Cook 2002). The main advantages of this technique are that the measurement of water
34 table depth in a well is easy and accurate and that the method is direct (not as in chemical techniques).
35 The attractiveness of the WTF method lies in its simplicity and ease of use compared to other
36 quantitative methods such as numerical models which require comprehensive data sets on aquifer
37 parameters and flow fields (Sanford 2002). No assumptions are made on the mechanisms by which
38 water infiltrates vertically through the unsaturated zone; hence, the presence of preferential flow
39 paths (localized recharge) within the unsaturated zone does not restrict its application. Because the
40 water level measured in an observation well is representative of a given area (at least several square
41 meters), the WTF method can be viewed as an integrated approach and less a point measurement
42 than those methods that are based on data in the unsaturated zone (Healy and Cook 2002).
43 Applications at the watershed scale, using observation well networks have been used in numerous
44 studies in various geological and climatic conditions (Maréchal et al. 2006; Misstear et al. 2009; Jassas
45 and Merkel 2014; Ala-Aho et al. 2015; Chemingui et al. 2015; Labrecque et al. 2020). This method has

46 been implemented through recent development of the tool ESPERE for computing recharge (Lanini et
47 al. 2016; Lanini and Caballero 2021).

48 The WTF method requires knowledge of the specific yield of the aquifer (Sophocleous 1991) and the
49 identification of a well where the groundwater level can be monitored . Wells should be located so
50 that the water levels they monitor are representative of the aquifer as a whole (Healy and Cook 2002).

51 In this Note, the location of observation wells is investigated by means of basic equations governing
52 groundwater flow in an unconfined aquifer under simplified assumptions. Storage and discharged
53 volumes during WTF application are described using analytical equations. The analysis provides simple
54 and practical criteria to quantify, on one hand, the accuracy on natural recharge evaluations based on
55 water level measurements in existing observation wells and, on the other hand, to define the ideal
56 position where a new well should be drilled in order to improve the accuracy of the estimation.

57 **2. Problem statement**

58 Consider an unconfined, homogeneous and isotropic aquifer uniformly recharged from above, and
59 with an impervious bottom, as illustrated in Figure 1. This aquifer is connected to a stream, at $x = 0$,
60 where a constant piezometric level $h(0,t) = h_0$ is specified. Orthogonally away from the stream, at $x =$
61 L_0 , the aquifer is limited by a groundwater divide acting as a no-flow boundary, with a groundwater
62 level $h(L_0,t) = H(t)$ resulting from the recharge inflow. During a certain period of time T , the aquifer
63 infiltrates the uniform and constant recharge rate R , while it exfiltrates the transient discharge rate
64 $Q(t)$ into the stream. This one-dimensional model corresponds to those previously studied by (Erskine
65 and Papaioannou 1997; Cuthbert 2010).

66 If this period of time T is sufficiently long, the water table will reach a steady-state with $h(L_0) = H_0$ at
67 the groundwater divide and takes the classical elliptical shape,

$$68 \quad h(x) = \sqrt{h_0^2 + \frac{R}{K}(2L_0x - x^2)} = \sqrt{h_0^2 + \frac{(H_0^2 - h_0^2)}{L_0^2}(2L_0x - x^2)} \quad (1)$$

69 yielding the discharge rate

$$70 \quad Q = -Kh_0 \frac{\partial h(x)}{\partial x} \Big|_{x=0} = \frac{K}{L_0} (H_0^2 - h_0^2) = RL_0 \quad (2)$$

71 while the volume stored in the aquifer is

$$72 \quad V = S_y \int_0^{L_0} h(x) dx = \frac{S_y L_0}{2} \left(h_0 + H_0 \frac{\operatorname{asin}\left(\sqrt{1 - \frac{h_0^2}{H_0^2}}\right)}{\sqrt{1 - \frac{h_0^2}{H_0^2}}} \right) \quad (3)$$

73 where K is the hydraulic conductivity and S_y the specific yield of the aquifer. In particular cases where
74 $h_0/H_0 \ll 1$, this volume approaches $V = S_y \pi L_0 H_0/4$, corresponding to a quarter of the ellipse defined
75 by the half-axes L_0 and H_0 .

76 In a transient situation under a constant recharge, based on concepts similar to those presented in
77 drainage engineering textbooks, and those developed, tested and verified in works such as (Perrochet
78 and Musy 1992) and (Perrochet 2005) in two very different contexts, a simplified approach of excellent
79 accuracy is now enforced as an alternative to the exact, but cumbersome solution of the non-linear
80 transient equation governing the process. Assuming that the transient evolution of the groundwater
81 level can be treated as successive steady-state snapshots of elliptical shape, one may replace H_0 by
82 $H(t)$ in equations (1) - (3), namely

$$83 \quad h(x, t) = \sqrt{h_0^2 + \frac{(H(t)^2 - h_0^2)}{L_0^2} (2L_0x - x^2)} \quad (4)$$

84 and

$$85 \quad Q(t) = \frac{K}{L_0} (H(t)^2 - h_0^2) \quad (5)$$

86 and

87

$$V(t) = \frac{S_y L_0}{2} \left(h_0 + H(t) - \frac{\operatorname{asin}\left(\sqrt{1 - \frac{h_0^2}{H(t)^2}}\right)}{\sqrt{1 - \frac{h_0^2}{H(t)^2}}} \right) \quad (6)$$

88 To find out $H(t)$, one applies the global mass balance equation, accounting for the storage variation
 89 (shown in dark blue in Figure 1), as expressed in equation (7).

90

$$Q(t) = -\frac{\partial V(t)}{\partial t} + RL_0 \quad (7)$$

91 Combined with equations (5) and (6), this equation allows for solving the unknown function $H(t)$, the
 92 solution of which is

93

$$H(t) = \sqrt{h_0^2 + \frac{R}{K} L_0^2} \left(\frac{H_0 + \sqrt{h_0^2 + \frac{R}{K} L_0^2} \tanh\left(\frac{4K}{\pi \mu L_0^2} \sqrt{h_0^2 + \frac{R}{K} L_0^2} t\right)}{H_0 \tanh\left(\frac{4K}{\pi S_y L_0^2} \sqrt{h_0^2 + \frac{R}{K} L_0^2} t\right) + \sqrt{h_0^2 + \frac{R}{K} L_0^2}} \right) \quad (8)$$

94 with H_0 as initial condition.

95 Let consider a period of time with a duration T , during which the WTF method can be applied. The
 96 natural recharge (infiltration to the water table) during the interval T can be estimated with

97

$$R = \frac{V_Q(T) + V_S(T)}{L_0 T} \quad (9)$$

98 where V_Q is the volume discharged and V_S the volume entering storage or leaving storage
 99 (replenishment of groundwater storage) during the time period T .

100 Combining equation (8), with equations (5) and (6), the volume discharged V_Q during this interval is:

101

$$V_Q(T) = \int_0^T Q(t) dt = L_0 R T \left(\frac{\sqrt{h_0^2 + \frac{R}{K} L_0^2} + \left(H_0 + \frac{\pi S_y}{4RT} (H_0^2 - h_0^2 - \frac{R}{K} L_0^2) \right) \tanh\left(\frac{4K}{\pi S_y L_0^2} \sqrt{h_0^2 + \frac{R}{K} L_0^2} T\right)}{\sqrt{h_0^2 + \frac{R}{K} L_0^2} + H_0 \tanh\left(\frac{4K}{\pi S_y L_0^2} \sqrt{h_0^2 + \frac{R}{K} L_0^2} T\right)} \right) \quad (10)$$

102 On the other hand, the volume entering storage or leaving storage V_S during the time period T can now
 103 be obtained by

104

$$V_S(T) = V(T) - V(0) \quad (11)$$

105 applying equation (6).

106 In practice, it is suggested to estimate the discharge rate $Q(t)$ – and hence $V_Q(T)$ – by direct
107 measurements in the river draining the aquifer. Applying the WTF method, the variation of water
108 storage $V_S(T)$ in the aquifer can be inferred from regionalized piezometric data, when the latter are
109 available. If not, and as shown below, a single piezometer may also be able to yield a correct value
110 provided its location is carefully selected.

111 3. Results

112 In the present configuration, identifying the location where the local water table fluctuation is
113 representative of the mean fluctuation over the entire domain, requires the solution for X of

$$114 \quad h(X, t) = \frac{1}{L_0} \int_0^{L_0} h(x, t) dx \quad (12)$$

115 using equation (4), where X is the distance from the stream. The exact solution to this problem depends
116 on the ratio $h_0/H(t)$, which evolves during the time interval, however over a relatively narrow range,
117 bounded by the two following cases:

$$118 \quad \text{- elliptical piezometric surface: } h_0/H(t) \rightarrow 0, X = \left(1 - \sqrt{1 - \left(\frac{\pi}{4}\right)^2}\right) L_0 = 0.381 L_0$$

$$119 \quad \text{- rather horizontal piezometric surface: } h_0/H(t) \rightarrow 1, X = \left(1 - \frac{1}{\sqrt{3}}\right) L_0 = 0.423 L_0$$

120 Interestingly, this distance X is not dependant on the hydrodynamic parameters of the aquifer K and S_y

121 An excellent approximation of this location where the water table fluctuation is representative of the
122 mean fluctuation over the aquifer is therefore obtained with an average coefficient, yielding the estimator

123 \bar{V}_S for the groundwater volume stored during the interval T as

$$124 \quad \bar{V}_S(T) = S_y L_0 (h_{\text{obs}}(X, T) - h_{\text{obs}}(X, 0)), X = 0.4 L_0 \quad (13)$$

125 with only negligible residual errors, as shown on Figure 2 (green line), and with no required prior knowledge
 126 of h_0 , H_0 or $H(t)$. In the above equation h_{obs} stands for the piezometric levels observed, respectively, at $t =$
 127 T and initially at $t = 0$.

128 In the case that an observation well is not located at an ideal position, the relative error on the estimation
 129 of groundwater volumes depends on both the ratio $h_0 / H(t)$ and the position of the observation well X .

$$130 \quad \text{Err}(X) = \frac{\overline{V_s}(X) - V_s(X)}{\overline{V_s}(X)} = 1 - \frac{\frac{h_0}{H} + \frac{\text{asin}\left(\sqrt{1 - \frac{h_0^2}{H^2}}\right)}{\sqrt{1 - \frac{h_0^2}{H^2}}}}{2 \sqrt{\frac{h_0^2}{H^2} + \left(1 - \frac{h_0^2}{H^2}\right) \left(\frac{2X}{L_0} - \frac{X^2}{L_0^2}\right)}} \quad (14)$$

131 This error is maximum for $h_0 / H(t) = 0$ (corresponding to a quite common case where the stream depth is
 132 negligible compared to the elliptical piezometric head into the aquifer), in which case it is expressed by:

$$133 \quad \text{Err}(X) = 1 - \frac{\pi}{4 \sqrt{\frac{2X}{L_0} - \left(\frac{X}{L_0}\right)^2}} \quad (15)$$

134 Figure 2 provides the relative error on groundwater volumes V_s estimated with $h_{\text{obs}}(X, t)$ as a function of
 135 the ratio $h_0 / H(t)$ for different locations of the observation well ($X / L_0 = 0.1, 0.2, \dots, 0.381$ (red), 0.4
 136 (green), 0.423 (blue), 0.5, ..., 0.9). Positive values indicate an overestimation of the stored volume while
 137 negative values indicate an underestimation. Logically, the stored volume is underestimated for locations
 138 close to the stream as water table fluctuations are reduced by the fixed head boundary condition at the
 139 stream. In contrast, the stored volume is overestimated close to the water divide where water table
 140 fluctuations are higher than the average.

141 It is observed that the error on V_s fluctuates between 0 for a flat piezometric surface ($h_0 / H(t) \rightarrow 1$)
 142 and maximum values for the elliptical surface ($h_0 / H(t) \rightarrow 0$).

143 Figure 3 illustrates the evolution, for the worst case (elliptical piezometric surface), of the error on
 144 computed V_s using the WTF method for different locations of the observation well, ranging from $X = 0.1 L_0$

145 (close to the stream) L_0 to $X = 0.8 L_0$ (close to the water divide). Error is vanishing around $X = 0.4 L_0$ while
146 the volume is highly underestimated (up to 80%) for $X = 0.1 L_0$ and gently overestimated (about 20%) at X
147 $= L_0$. Equations (14) and (15) can be used to correct the estimator
148 \bar{V}_S for the groundwater volume stored during the interval T obtained using WTF Equation (13) with water
149 level measurements from an observation well that is not properly located.

150

151 **4. Conclusion**

152 The present analysis provides several guidelines for the implementation of the WTF method for recharge
153 estimation under the simplified conditions described here (constant and uniform recharge, homogeneous
154 aquifer, one-dimensional flow to the stream). First, the perfect position for an observation well is located
155 at four tenths of the distance ($X = 0.4 L_0$) from the stream to the aquifer divide. Beyond this distance, the
156 application of the method leads to an overestimation of the natural recharge ranging from 0 at the ideal
157 position ($X = 0.4 L_0$) to 21 % at the aquifer divide ($X = L_0$). For observation wells located close to the stream,
158 the hydraulic condition imposed by the stream leads to an underestimation of the natural recharge estimate
159 ranging from 0 at the ideal position ($x = 0.4 L_0$) to 80 % for $X = 0.1 L_0$ and even more closer to the stream.
160 These results are generalizable as they are not dependant on the diffusivity or other parameters of the
161 aquifer. They indicate that an observation well should always be located at least farther, if not in the vicinity
162 of $X = 0.4 L_0$, for an accurate estimation of the natural recharge using the WTF method. The error curves
163 and equations presented allow one to estimate potential errors on stored groundwater volumes and
164 possibly correct the time series located too far away from the ideal position $X = 0.4 L_0$.

165 **Acknowledgments**

166 This study has been funded by BRGM and Agence de l'Eau Rhône Méditerranée Corse (AERMC) in the
167 framework of their research agreement.

168 On behalf of all authors, the corresponding author states that there is no conflict of interest.

169

170 **References**

- 171 Ala-Aho P, Rossi PM, Kløve B (2015) Estimation of temporal and spatial variations in groundwater
172 recharge in unconfined sand aquifers using Scots pine inventories. *Hydrol Earth Syst Sci* 19:1961–
173 1976. <https://doi.org/10.5194/HESS-19-1961-2015>
- 174 Chemingui A, Sulis M, Paniconi C (2015) An assessment of recharge estimates from stream and well
175 data and from a coupled surface-water/groundwater model for the des Anglais catchment, Quebec
176 (Canada). *Hydrogeol J* 23:1731–1743. <https://doi.org/10.1007/s10040-015-1299-1>
- 177 Cuthbert MO (2010) An improved time series approach for estimating groundwater recharge from
178 groundwater level fluctuations. *Water Resour Res* 46:9515. <https://doi.org/10.1029/2009WR008572>
- 179 Erskine AD, Papaioannou A (1997) The use of aquifer response rate in the assessment of
180 groundwater resources. *J Hydrol* 202:373–391. [https://doi.org/10.1016/S0022-1694\(97\)00058-9](https://doi.org/10.1016/S0022-1694(97)00058-9)
- 181 Healy RW, Cook PG (2002) Using groundwater levels to estimate recharge. *Hydrogeol J* 10:91–109.
182 <https://doi.org/10.1007/s10040-001-0178-0>
- 183 Jassas H, Merkel B (2014) Estimating Groundwater Recharge in the Semiarid Al-Khazir Gomal Basin,
184 North Iraq. *Water* 2014, Vol 6, Pages 2467-2481 6:2467–2481. <https://doi.org/10.3390/W6082467>
- 185 Labrecque G, Chesnaux R, Boucher MA (2020) Water-table fluctuation method for assessing aquifer
186 recharge: application to Canadian aquifers and comparison with other methods. *Hydrogeol J* 28:521–
187 533. <https://doi.org/10.1007/s10040-019-02073-1>
- 188 Lanini S, Caballero Y (2021) ESPERE, a Tool for Multimethod Aquifer Recharge Estimation: What’s
189 New with Version 2? *Groundwater* 59:5–6
- 190 Lanini S, Caballero Y, Seguin J-J, Maréchal J-C (2016) ESPERE-A Multiple-Method Microsoft Excel
191 Application for Estimating Aquifer Recharge. *Groundwater* 54:155–156.
192 <https://doi.org/10.1111/gwat.12390>

193 Maréchal JC, Dewandel B, Ahmed S, Galeazzi L, Zaidi FK (2006) Combined estimation of specific yield
194 and natural recharge in a semi-arid groundwater basin with irrigated agriculture. *J Hydrol* 329:281–
195 293. <https://doi.org/10.1016/j.jhydrol.2006.02.022>

196 Misstear BDR, Brown L, Johnston PM (2009) Estimation of groundwater recharge in a major sand and
197 gravel aquifer in Ireland using multiple approaches. *Hydrogeol J* 17:693–706.
198 <https://doi.org/10.1007/S10040-008-0376-0/TABLES/7>

199 Perrochet P (2005) A simple solution to tunnel or well discharge under constant drawdown.
200 *Hydrogeol J* 13:886–888. <https://doi.org/10.1007/s10040-004-0355-z>

201 Perrochet P, Musy A (1992) A simple formula to calculate the width of hydrological buffer zones
202 between drained agricultural plots and nature reserve areas. *Irrig Drain Syst* 6:69–81.
203 <https://doi.org/10.1007/BF01102867>

204 Sanford W (2002) Recharge and groundwater models: An overview. *Hydrogeol J* 10:110–120.
205 <https://doi.org/10.1007/s10040-001-0173-5>

206 Scanlon BR (2011) *Estimating groundwater recharge*. Cambridge University Press, Cambridge

207 Scanlon BR, Healy RW, Cook PG (2002) Choosing appropriate techniques for quantifying groundwater
208 recharge. *Hydrogeol J* 2002 10:18–39. <https://doi.org/10.1007/S10040-001-0176-2>

209 Sophocleous MA (1991) Combining the soilwater balance and water-level fluctuation methods to
210 estimate natural groundwater recharge: Practical aspects. *J Hydrol* 124:229–241.
211 [https://doi.org/10.1016/0022-1694\(91\)90016-B](https://doi.org/10.1016/0022-1694(91)90016-B)

212 **FIGURE CAPTIONS :**

213 Figure 1 : vertical 2D cross section of an aquifer of length L_0 under recharge (R) and discharge (Q)
214 conditions

215 Figure 2 : relative error on the stored groundwater volume V_s computed using water level fluctuation
216 measured at varying distances from the stream ($X= 0.1, 0.2, \dots, 0.381$ (red), 0.4 (green), 0.423 (blue),
217 $0.5, \dots, 0.9$)

218 Figure 3 : vertical 2D cross section of an aquifer of length L_0 under recharge R and discharge Q at a
219 stream ($x = 0$). Location of several observation wells at $x = 0.1 L_0$, $x = 0.4 L_0$ and $x = 0.8 L_0$. Illustration
220 of the error on stored volume V_s computed as a function of the distance of the observation well to
221 the stream.