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Accidental eccentricity in symmetric buildings due to wave passage effects arising from near-fault pulse-like ground motions

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SUMMARY

This article investigates the characteristics of the accidental eccentricity in symmetric buildings due to torsional response arising from wave passage effects in the near-fault region. The soil–foundation–structure system is modeled as a symmetric cylinder placed on a rigid circular foundation supported on an elastic halfspace and subjected to obliquely incident plane SH waves simulating the action of near-fault pulse-like ground motions. The translational response is computed assuming that the superstructure behaves as a shear beam under the action of translational and rocking base excitations, whereas the torsional response is calculated using the mathematical formulation proposed in a previous study. A broad range of properties of the soil–foundation–structure system and ground motion input are considered in the analysis, thus facilitating a detailed parametric investigation of the structural response. It is demonstrated that the normalized accidental eccentricity is most sensitive to the pulse period (Tₚ) of the near-fault ground motions and to the uncoupled torsional-to-translational fundamental frequency ratio (Ω) of the structure. Furthermore, the normalized accidental eccentricities due to simplified pulse-like and broadband ground motions in the near-fault region are computed and compared against each other. The results show that the normalized accidental eccentricity due to the broadband ground motion is well approximated by the simplified pulse for longer period buildings, while it is underestimated for shorter period buildings. For symmetric buildings with values of Ω commonly used in design practice, the normalized accidental eccentricity due to wave passage effects is less than the typical code-prescribed value of 5%, except for buildings with very large foundation radius. Copyright © 2017 John Wiley & Sons, Ltd.

1. INTRODUCTION

Earthquake-induced torsion in buildings can be classified into ‘natural’ and ‘accidental’. The natural (or inherent) torsion is induced by the geometrical separation of the centers of mass and stiffness, resulting in coupled lateral–torsional response. On the other hand, the accidental torsion is associated with torsional vibrations generated by spatially varying seismic excitations, uncertainties in the distributions of stiffness and mass, and other factors that are not explicitly considered in the analysis and design of buildings.

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Most building codes require the effects of accidental torsion to be considered by applying the equivalent static lateral force at a distance \( e_a \) from the center of stiffness, resulting in accidental torsional moments that are typically resisted by increasing the design shear forces in the lateral resisting system. The quantity \( e_a \), also known as design accidental eccentricity, is usually defined in building codes as a fraction (e.g. 5% or 10%) of the building dimension. Several studies have investigated the effects of spatially varying seismic excitations (resulting from ground motion incoherence and wave passage) on the torsional response of buildings and evaluated the adequacy of the accidental eccentricity prescribed by building codes, but only a few are summarized in this section. The interested reader is referred to Anagnostopoulos et al. [1] for a comprehensive literature review of accidental torsion in buildings.

Newmark [2] made the first rational attempt to investigate the torsional response of symmetric buildings due to base rotations arising from earthquake waves propagating with a constant velocity. It was concluded that the ratio of the design accidental eccentricity to the long plan dimension of the building varies directly with the fundamental frequency of vibration and with the transit time of the earthquake wave motion across the long plan dimension. Luco [3] presented the mathematical formulation of the torsional response of a symmetric, elastic structure placed on a surface-supported foundation under the action of obliquely incident plane SH waves, while accounting for soil–structure interaction (SSI) effects. It was demonstrated that large displacements associated with torsional response may be generated even for symmetric structures. Luco and Sotiropoulos [4] investigated the effects of wave passage on the torsional response of a square foundation and showed that an accidental eccentricity of 5% is sufficient to account for the effects of wave passage on structures with a fundamental frequency less than 5 Hz and foundation length in the direction of propagation less than 40 m. For structures with fundamental frequency and foundation length not satisfying these conditions or for structures located at short distances from the seismic source, it was determined that the accidental eccentricity can be greater than 5%.

Morgan et al. [5] investigated the translational and torsional response of buildings arising from traveling waves using strong motion data. They showed that the effects of induced rotations on structures are not important for fundamental frequencies less than 1 Hz and the 5% accidental eccentricity specified in building codes is adequate. Wu and Leyendecker [6] examined the torsional response of a single-storey system subjected to an incident SH wave and concluded that the effects of accidental eccentricity on the dynamic response of symmetric buildings cannot be ignored. De la Llera and Chopra [7] extracted base torsional excitations, associated with spatially varying ground motions, from translational recordings at the foundation level of actual buildings. They demonstrated that the increase in building displacements resulting from accidental torsion due to the extracted torsional excitations may be large for short-period (i.e. less than 0.5 s) and torsionally flexible systems. In addition, the computed accidental eccentricities were found to be significantly smaller than the typical code values, except for buildings with very long plan dimensions (i.e. greater than 50 m).

Heredia-Zavoni and Barranco [8] examined the torsional response of symmetric structural systems subjected to spatially varying ground motions due to ground motion incoherence, wave passage, and local soil conditions. It was demonstrated that the accidental eccentricity may exceed the standard code-prescribed values of 5 and 10% depending on the period and aspect ratio of the system, the local soil conditions, and the times for the seismic waves to travel across the base of the system. Heredia-Zavoni and Leyva [9] investigated the effects of ground motion incoherence and wave passage on the torsional response of three-dimensional, multi-storey, multi-span, symmetric, linear elastic buildings. They showed that the use of a base shear along with the design accidental eccentricity specified in the building code for Mexico City may result in considerably high estimates of shear force in corner columns of structures on firm soil; however, for soft soil conditions, the shear force in corner columns may be underestimated, especially for short-period buildings with long plan dimensions. Aviles and Suarez [10] evaluated the natural and accidental torsion in a single monosymmetric structure with consideration of SSI. The structure was placed on a rigid foundation embedded into an elastic halfspace, whereas the earthquake excitation was assumed to be composed of non-vertically incident SH waves. By considering different combinations of the system parameters and an ensemble of ground motion records, it was demonstrated that the computed accidental
eccentricity does not exceed the smallest codified value of 5% except for surface-supported structures with long plan dimensions.

In a recent study, Cao et al. [11] investigated the effects of wave passage on the torsional response of symmetric buildings in the near-fault region. The soil–foundation–structure system was modeled as a symmetric cylinder placed on a rigid circular foundation supported on an elastic halfspace and subjected to obliquely incident plane SH waves simulating the action of near-fault pulse-like ground motions. A detailed parametric analysis of the maximum relative twist between the top and the base of the superstructure was performed to identify the parameters of the seismic excitation input and soil–foundation–structure system that control the torsional response. It was shown that large torsional response is observed when a key parameter of the near-fault ground motions referred to as ‘pulse period’ is close to the fundamental torsional period of the structure.

Building upon the results of Cao et al. [11], this article aims to further evaluate the increase in building displacements due to the torsional response arising from wave passage effects in the near-fault region. This is achieved by comparing the accidental eccentricity, estimated from the translational and torsional components of the building response, with code-prescribed values. The translational response is computed using a closed-form expression derived in the present study assuming that the superstructure behaves as a shear beam under the action of translational and rocking base excitations, whereas the torsional response is calculated using the mathematical formulation proposed by Luco [3]. A broad range of properties of the soil–foundation–structure system and ground motion input are considered in the analysis, thus facilitating a detailed parametric investigation. Finally, the effectiveness of idealized pulse models to accurately estimate the accidental eccentricity for symmetric buildings arising from wave passage effects in the near-fault region is assessed by estimating and comparing the accidental eccentricities due to simplified pulse-like and broadband ground motions.

2. NEAR-FAULT PULSE-LIKE GROUND MOTIONS

Forward rupture directivity and/or permanent translation (fling) are the two primary physical processes that give rise to pulse-like ground motions in the near-fault region. In this study, the mathematical model proposed by Mavroeidis and Papageorgiou [12] is adopted to describe the coherent component of the near-fault ground motions. The formulation of the mathematical model for the ground velocity pulses is the product of a harmonic oscillation and a bell-shaped function:

\[
v(t) = \begin{cases} 
\frac{A}{2} \left[ 1 + \cos \left( \frac{2\pi f_0^2}{\gamma} (t - t_0) \right) \right] \cos \left[ 2\pi f_P(t - t_0) + \phi \right], & t_0 - \frac{\gamma}{2f_0} \leq t \leq t_0 + \frac{\gamma}{2f_0} \\
0, & \text{otherwise} 
\end{cases} \tag{1}
\]

where \( A \) controls the amplitude of the signal, \( f_0 \) is the prevailing frequency of the signal, \( \phi \) is the phase of the amplitude-modulated harmonic, \( \gamma \) is a parameter that defines the oscillatory character of the signal, and \( t_0 \) specifies the epoch of the envelope’s peak. The pulse period \( T_P \) is defined as the inverse of the prevailing frequency \( f_0 \), that is: \( T_P = 1/f_0 \). Mavroeidis and Papageorgiou [12] also derived a closed-form expression for the Fourier transform \( V(\omega) \) of the velocity signal provided by Equation (1) as a function of \( A, f_0, \gamma, \) and \( \phi \). The Fourier transform \( D(\omega) \) of the corresponding displacement signal can be expressed as \( D(\omega) = V(\omega)/i(\omega) \).

Empirical relationships and practical guidelines have been proposed [12–14] for the selection of the input parameters of the mathematical model of Equation (1). For interplate tectonic regimes and under the requirement of self-similarity, the pulse period \( T_P \) is related to the earthquake magnitude \( M_W \) through the following equation [12, 14]:

\[
\log T_P = -2.9 + 0.5M_W \tag{2}
\]

The amplitude of the near-fault velocity records has been found to be a fairly stable parameter with a value of 100 cm/s effectively representing the peak ground velocity within a few kilometers from the
fault regardless of the earthquake magnitude [12]. Attenuation relationships for peak ground velocity with rupture distance have also been proposed by [13, 14]. Parameter $\gamma$ varies from a value slightly larger than 1 up to a maximum value of 3, whereas the phase angle $\nu$ ranges from 0 to 360° [12, 13]. Probability density functions for $\gamma$ and $\nu$, assuming that these two parameters are normally distributed, have also been proposed in [13].

Following Cao et al. [11], $T_P$ is assumed to range from 0.5 to 10 s, which corresponds to $M_W$ between 5.2 and 7.8 according to Equation (2). Parameter $A$ is set equal to 100 cm/s for sites located within a few kilometers from the fault. The effect of $\gamma$ is taken into account by considering four characteristic values: 1.0, 1.5, 2.0, and 3.0. Finally, parameter $\nu$ is set equal to 90° to ensure that the displacement offset vanishes for all synthetic motions in agreement with the waveform characteristics of pure forward directivity motions (see discussion in [11]).

3. SOIL–FOUNDATION–STRUCTURE SYSTEM

Figure 1a illustrates the soil–foundation–structure model adopted in this study (modified after [3]) to evaluate the effects of wave passage on the accidental eccentricity of symmetric, elastic buildings subjected to near-fault pulse-like ground motions. The superstructure is modeled as a uniform elastic bar of height $H$ and radius $a$. The foundation is represented by a surface-supported rigid circular disc of negligible thickness and the same radius as the superstructure. Parameters $m_b$ and $m_0$ denote the masses of the superstructure and the foundation, respectively, whereas the corresponding mass moments of inertia about the $z$ axis are $I_b$ and $I_0$. In addition, $J_0$ represents the mass moment of inertia of the foundation about the $y$ axis. The superstructure is characterized by uncoupled fixed-base fundamental translational and torsional frequencies $\omega_x$ and $\omega_\theta$, respectively, whereas the corresponding hysteretic damping factors are denoted by $\xi_x$ and $\xi_\theta$. The soil is assumed to be an elastic, homogeneous, and isotropic halfspace with shear wave velocity $\beta$, shear modulus $G$, Poisson’s ratio $\sigma$, and density $\rho_s$.

As shown in Figure 1a, the excitation of the system is represented by an obliquely incident plane SH wave propagating at an angle $\Theta$ with respect to the $y$ axis (i.e. angle of incidence), whereas the particle motion is parallel to the $x$ axis. Under this type of excitation, the response of the foundation–superstructure system can be decomposed into two parts: translation and rocking in the $x$–$z$ plane (Figure 1b) and torsion about the $z$ axis (Figure 1c). Because the superstructure and the foundation are both symmetric, the translational-rocking and torsional responses are uncoupled and thus can be studied independently. In order to model realistic building configurations and site conditions, 40 actual buildings with distinct characteristics are considered in this study. The building properties,
obtained from the empirical investigation of SSI effects conducted by Stewart et al. [15], have been summarized in Table 1 of Cao et al. [11] and thus are not discussed further herein for the sake of brevity.

Even though the soil–foundation–structure model considered in this study does not account for the effects of nonlinear response and foundation flexibility, it is considered to be appropriate for the following reasons: (i) the response of buildings to strong earthquake motions, although nonlinear, may be described by linear models with appropriate values of period and equivalent damping, provided that the structural yielding is not excessive and is well distributed over the structure [16]. It is clear that more realistic models that take into account the detailed building geometry and the nonlinear response are required to capture more accurately the behavior of the soil–foundation–structure system for design purposes; (ii) the near-fault pulse-like ground motions considered in this study are associated with wavelengths that are significantly greater than the size of the footprint of a reasonably-sized structure, thus implying that the foundation flexibility would not affect the structural response considerably; and (iii) the selected model provides the analytical tools to quantify the impact of variations in the input parameters of the seismic excitation and soil–foundation–structure system on structural response.

4. TRANSLATIONAL RESPONSE DUE TO SIMPLIFIED NEAR-FAULT PULSE-LIKE GROUND MOTIONS

4.1. Transfer functions

4.1.1. Mathematical formulation. The mathematical formulation that follows was originally developed, and preliminary results were presented by Meza-Fajardo and Papageorgiou [17]. The seismic excitation depicted in Figure 1a is represented by an obliquely incident plane SH wave of amplitude $u_{g0}/2$ and steady-state frequency $\omega$ expressed by

$$U_{gx}^i(x,y,z) = \frac{u_{g0}}{2} e^{i\omega(t-\frac{y}{C_0}\cos\theta-\frac{z}{C_0}\sin\theta)}$$

(3)

In the absence of the rigid foundation, the incident wave of Equation (3) and its total reflection on the free surface combine to generate the free-field motion

$$U_{gx}^{i+r}(x,y,z) = u_{g0}e^{i\omega(t-\frac{y}{C_0}\cos\theta)}\cos\left(\frac{\omega z}{\beta}\sin\theta\right)$$

(4)

Figure 1b illustrates the response of the foundation–superstructure system in the $x$–$z$ plane under the action of the incident wave of Equation (3). The foundation response consists of a translational horizontal component $U_0$ along the $x$ axis and a rocking component $\Psi_0$ about the $y$ axis, both defined with respect to the position of the foundation prior the occurrence of seismic disturbances. On the other hand, the response of the superstructure is described by a horizontal translation $\zeta\Psi_0$ at level $z$ relative to its base, which is in addition to the horizontal translation $z\Psi_0$ caused by foundation rocking.

The response of the foundation, which consists of horizontal translation $U_0$ and rocking $\Psi_0$, may conveniently be written in the form of (e.g. [18])

$$\begin{cases} U_0 \\ a\Psi_0 \end{cases} = \begin{cases} U_0^* \\ a\Psi_0^* \end{cases} + \begin{cases} U_s \\ a\Psi_s \end{cases}$$

(5)

where $U_0^*$ and $\Psi_0^*$ are the foundation input motions obtained by evaluating the response of the rigid massless foundation to the incident SH wave in the absence of the superstructure, and $U_s$ and $\Psi_s$ are additional motions of the foundation associated with the deformation of the soil caused by the force and moment that the foundation exerts on the soil. For conciseness, the time factor $e^{i\omega t}$ is omitted.
both from Equation (5) and from the mathematical formulation presented in the remainder of this section.

The translational component \( U_0^* \) of the foundation input motion for the soil–foundation–structure system shown in Figure 1a can be calculated by multiplying a frequency-dependent function \( S_{xx} \) by the free-field motion of Equation (4) evaluated at \((y,z) = (0,0)\):

\[
U_0^* = S_{xx} u_{g0}
\]  

(6)

Luco and Mita [19] have provided values of \( S_{xx} \) in tabular form as a function of the dimensionless frequency \( a_0 = \omega_0 f_0 \) for \( \beta c_{app} = 0.1, 0.2, 0.3, 0.4, 0.5 \) and \( \sigma = 0.33, 0.45 \), where \( c_{app} \) is the wave apparent velocity (i.e. \( \beta c_{app} = \beta \cos \Theta \)). These \( \beta c_{app} \) values correspond to angles of incidence \( \Theta \) ranging from 60.0° to 84.3°, which are deemed sufficient to characterize the incident angles of shear waves in the near-fault region (e.g. [11, 20]). In addition, the rocking component \( \Psi_0^* \) of the foundation input motion has been found to be very small for flat foundations excited by SH waves and can be neglected for all practical purposes [21].

On the other hand, the additional motions \( U_s \) and \( \Psi_s \) are related to the force \( V_s \) and moment \( M_s \) that the foundation exerts on the soil as follows

\[
\begin{bmatrix}
V_s \\
M_s \\
\frac{a}{s}
\end{bmatrix} = K(\omega) \begin{bmatrix}
U_s \\
0 \\
0
\end{bmatrix} = \begin{bmatrix} K_{HH} & K_{HM} \\ K_{MH} & K_{MM} \end{bmatrix} \begin{bmatrix}
U_s \\
0
\end{bmatrix}
\]  

(7)

where \( K(\omega) \) is the frequency-dependent impedance matrix. The terms \( K_{HH}, K_{HM} = K_{MH}, \) and \( K_{MM} \) correspond to the normalized horizontal, coupling, and rocking impedance functions, for which tabular data, as a function of \( a_0 = \omega_0 f_0 \) for \( \sigma = 0.33, 0.45 \), have also been provided by [19].

Assuming that the superstructure is modeled as a uniform shear beam, an approach that has widely been used by earthquake engineers to study the seismic response of buildings (despite its limitation of only accounting for shear deformations), the equation of motion under harmonic excitation of foundation translation \( U_0 \) and rocking \( \Psi_0 \) is given by

\[
k(1 + 2i\xi_s) \frac{d^2 U(z)}{dz^2} + m\omega^2 U(z) = -m\omega^2 U_0 - m\omega^2 \Psi_0, \quad 0 \leq z \leq H
\]  

(8)

where \( U(z) \) is the translational response, \( k \) is the shear stiffness, \( m \) is the mass per unit length, and \( \xi_s \) is the hysteretic damping factor for translational vibrations. The fundamental translational frequency \( \omega_x \) of the fixed-base superstructure is

\[
\omega_x = \frac{\pi}{2H} \sqrt{\frac{k}{m}}
\]  

(9)

The translational response \( U(z) \) must satisfy the following boundary conditions

\[
U(z = 0) = 0, \quad V(z = H) = k(1 + 2i\xi_s) \frac{dU(z)}{dz} \bigg|_{z = H} = 0
\]  

(10a–b)

where \( V(z) \) is the shear force at level \( z \) and \( H \) is the height of the structure. The solution of Equation (8), subject to the boundary conditions expressed by Equation (10a–b), is

\[
U(z) = \left[ \cos(k_{bx}z) + \tan(k_{bx}H) \sin(k_{bx}z) - 1 \right] U_0 + \left[ \frac{\sin(k_{bx}z)}{k_{bx} \cos(k_{bx}H) - z} \right] \Psi_0
\]  

(11)

where
The base shear $V_b$ and base moment $M_b$ that the superstructure exerts on the foundation may then be evaluated as

$$V_b = k(1 + 2i\omega_k)\frac{dU(z)}{dz} \bigg|_{z=0} = \left(\frac{m\omega^2}{kbx}\right) \tan(kbxH) U_0 + \left(\frac{\sec(kbxH) - 1}{kbx}\right) \Psi_0$$

(13)

$$M_b = \int_0^H \frac{m\omega^2}{kbx} [U(z) + U_0 + z\Psi_0] dz = \left(\frac{m\omega^2}{kbx}\right) \left[\frac{\sec(kbxH) - 1}{kbx}\right] U_0 + \left(\frac{1}{kbx}\right) \tan(kbxH) - H \Psi_0$$

(14)

Equations (13) and (14) can properly be normalized and expressed in matrix form as

$$\begin{bmatrix} V_b \\ M_b/a \end{bmatrix} = \omega^2 \left(\frac{m_s}{kbxH}\right) \left(\frac{H}{a}\right) \begin{bmatrix} \left(\frac{H}{a}\right)^{-1} \tan(kbxH) & \sec(kbxH) - 1 \frac{kbxH}{} \\ \sec(kbxH) - 1 \frac{kbxH}{} & \left(\frac{H}{a}\right)^{-1} \tan(kbxH) - kbxH (kbxH)^2 \end{bmatrix} \begin{bmatrix} U_0 \\ a\Psi_0 \end{bmatrix}$$

(15)

where $m_s = mH$ is the mass of the superstructure.

By considering the coupling of superstructure and soil through the foundation, the dynamic force and moment equilibrium equations for the foundation can be written in matrix form as

$$-\omega^2 \begin{bmatrix} m_0 & 0 \\ 0 & (J_0/a^2) \end{bmatrix} \begin{bmatrix} U_0 \\ a\Psi_0 \end{bmatrix} = \begin{bmatrix} V_b \\ M_b/a \end{bmatrix} - \begin{bmatrix} V_s \\ (M_s/a) \end{bmatrix}$$

(16)

By substituting Equations (7) and (15) into Equation (16) and then combining with Equation (5), the response of the foundation ($U_0$ and $\Psi_0$) can be expressed in terms of the foundation input motions ($U_0^a$ and $\Psi_0^a$) as

$$\begin{bmatrix} U_0 \\ a\Psi_0 \end{bmatrix} = \begin{bmatrix} I_2 - \frac{\omega^2}{\omega_a^2} \left[ K_{HH} K_{MM} K_{SM} \right]^{-1} \begin{bmatrix} m_0 & 0 \\ 0 & (J_0/a^2) \end{bmatrix} + \left(\frac{m_s}{kbxH}\right) \left(\frac{H}{a}\right) \begin{bmatrix} \left(\frac{H}{a}\right)^{-1} \tan(kbxH) & \sec(kbxH) - 1 \frac{kbxH}{} \\ \sec(kbxH) - 1 \frac{kbxH}{} & \left(\frac{H}{a}\right)^{-1} \tan(kbxH) - kbxH (kbxH)^2 \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} U_0^a \\ a\Psi_0^a \end{bmatrix}$$

(17)

By using the relationship between shear wave velocity $\beta$ and shear modulus $G$ (i.e. $\beta = \sqrt{G/\rho_s}$) along with the definition of the dimensionless frequency $\omega_a$ (i.e. $\omega_a = \omega_0/\beta$), the expression $\omega^2/(Ga)$ appearing in Equation (17) may alternatively be written as $\omega_0^2/(\rho_s a^3)$. By substituting Equation (6) into Equation (17) and setting $\Psi_0^a = 0$ (based on the preceding discussion), the response of the foundation ($U_0$ and $\Psi_0$) can be expressed in terms of the free-field motion ($u_0$) as

$$\begin{bmatrix} U_0 \\ a\Psi_0 \end{bmatrix} = \begin{bmatrix} I_2 - \frac{\omega^2}{\omega_a^2} \left[ K_{HH} K_{MM} K_{SM} \right]^{-1} \begin{bmatrix} m_0 & 0 \\ 0 & (J_0/a^2) \end{bmatrix} + \left(\frac{m_s}{kbxH}\right) \left(\frac{H}{a}\right) \begin{bmatrix} \left(\frac{H}{a}\right)^{-1} \tan(kbxH) & \sec(kbxH) - 1 \frac{kbxH}{} \\ \sec(kbxH) - 1 \frac{kbxH}{} & \left(\frac{H}{a}\right)^{-1} \tan(kbxH) - kbxH (kbxH)^2 \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} u_0 \\ S_{\omega a}\rho \end{bmatrix}$$

(18)

where

$$a_x = a\omega_x/\beta = a_0\omega_x/\omega, \quad m_s = \rho_s a^3, \quad J_s = m_s a^2$$

(19a–c)
In Equations (18) and (19), \(a_s\) is the relative stiffness parameter for translational vibrations, and \(m_s\) and \(J_s\) are the 'soil mass' and 'soil moment of inertia' introduced to conveniently write Equation (18) in terms of dimensionless parameters.

Once the translational and rocking components of the foundation response are obtained from Equation (18), the translational response of the superstructure at any level \(z\) can readily be calculated using Equation (11). In particular, the translational response at the top of the superstructure \(U_H\) [i.e. \(U(z)\) at \(z = H\)] is provided by the following equation

\[
U_H = [\cos(k_{bs}H) + \tan(k_{bs}H) \sin(k_{bs}H) - 1]U_0 + \left[\frac{\tan(k_{bs}H)}{k_{bs}H} - 1\right] \left(\frac{H}{a}\right) (a\Psi_0) \tag{20}
\]

Equations (18) and (20) indicate that the frequency domain responses of the foundation (\(U_0\) and \(\Psi_0\)) and top of the superstructure (\(U_H\)), also known as transfer functions, may be described in terms of the following dimensionless parameters: normalized frequency \(\omega_0/\omega_x\), relative stiffness parameter \(a_s\) for translational vibrations, ratio of building height to foundation radius \(H/a\) (also known as slenderness ratio), mass ratios \(m_b/m_s\) and \(m_d/m_s\), moment-of-inertia ratio \(J_b/J_s\), hysteretic damping factor \(\xi_s\) for translational vibrations, function \(S_{xx}\), and normalized impedance functions \(K_{HH}, K_{HM}\), and \(K_{MM}\). Note that parameter \(k_{bs}H\), appearing in Equations (18) and (20), is also a dimensionless parameter which depends on \(\omega_0/\omega_x\) and \(\xi_s\) [see Equation (12)].

### 4.1.2. Input parameters.

In Table 1 of Cao et al. [11], the building height \(H\), the foundation radius \(a\), the fixed-base fundamental translational period \(T_x = 2\pi/\omega_x\), and the soil shear wave velocity \(\beta\) are provided for 40 buildings. Using this information, the slenderness ratio \(H/a\) and the relative stiffness parameter \(a_s = a_0/\omega_0\beta\) can readily be calculated for each building. The \(H/a\) values range from 0.3 to 6.9 with the larger values typically corresponding to tall buildings, whereas the \(a_s\) values are between 0.1 and 2.3; it is only Building A1 on soft soil (i.e. small \(a_s\)) and Building A17 with a large foundation radius that are characterized by \(a_s\) values of 3.4 and 4.4, respectively.

The mass ratios \(m_b/m_s\) and \(m_d/m_s\) and the moment-of-inertia ratio \(J_b/J_s\) are determined through the following steps. The mass of the superstructure \(m_b\) may be estimated from

\[
m_b = \rho_b HA \tag{21}
\]

where \(\rho_b\), \(H\), and \(A = \pi a^2\) are the mass density, height, and floor area of the superstructure. The mass ratio \(m_d/m_s\) may be determined by combining Equations (19b) and (21) resulting in \(m_d/m_s = \pi(\rho_d/\rho_s) (H/a)\). If \(a_0 = m_d/m_b\) denotes the foundation-to-superstructure mass ratio, \(m_d/m_s\) may be obtained by multiplying the mass ratio \(m_d/m_b\) by \(a_0\). Considering that the foundation is modeled as a circular disc, its moment of inertia about the \(y\) axis is calculated from \(J_0 = m_d a^2/4\). By combining the latter equation with Equation (19c), it follows that \(J_0/J_s = m_d/(4m_s)\). Therefore, dimensionless parameters \(m_b/m_s\), \(m_d/m_s\), and \(J_0/J_s\) may be determined for each building from the slenderness ratio \(H/a\) and typical values of soil density \(\rho_s\), mass density of the superstructure \(\rho_b\), and foundation-to-superstructure mass ratio \(a_0\). In this study, it is assumed that \(\rho_s = 1.95 \text{ Mg/m}^3\), \(\rho_b = 250 \text{ kg/m}^3\), and \(a_0 = 0.2\), consistent with typical properties of the soil–foundation–structure system (see discussion in [11]). Finally, the hysteretic damping factor \(\xi_s\) for translational vibrations is set equal to 4%.

### 4.1.3. Parametric analysis.

The characteristics of the transfer function of the translational response at the top of the superstructure, expressed by Equation (20), are investigated through a parametric analysis. Based on the discussion in Section 4.1.2, the selected values of the relative stiffness parameter and slenderness ratio are \(a_s = 0.1, 1.5, 3.0\) and \(H/a = 0.3, 1.0, 3.0, 6.0\). Note that large values of \(a_s\) typically represent very stiff buildings or very soft soils, whereas large values of \(H/a\) correspond to tall slender buildings. Parameters \(m_b/m_s\), \(m_d/m_s\), and \(J_0/J_s\) are then calculated using the selected values of \(H/a\) along with the fixed values of \(\rho_s\), \(\rho_b\), and \(a_0\) listed in Section 4.1.2. Subsequently, \(U_0\) and \(\Psi_0\) are computed from Equation (18) for the selected values of \(a_s\) and the tabulated values of \(S_{xx}\), \(K_{HH}, K_{HM}\), and \(K_{MM}\) provided by [19] for \(\beta c_{app} = 0.1, 0.3, \) and 0.5. Larger values of \(\beta c_{app}\) correspond to incident SH waves traveling in a more horizontal direction. Finally, \(U_H\) is calculated from Equation (20). It should be noted that the calculations for Poisson’s
ratios $\sigma = 0.33$ and $0.45$ produce very similar results; thus, $\sigma$ is fixed to 0.33 for the analysis results presented in the remainder of this article.

Figure 2 illustrates the variation of the modulus of translational response $|U_H/u_{g0}|$ at the top of the superstructure as a function of the normalized frequency $\omega/\omega_x$ for distinct values of $a_x$, $H/a$, and $\beta/c_{app}$. Inspection of Figure 2 reveals that the amplitude of the first peak response and the corresponding normalized resonance frequency decrease as $a_x$ increases from 0.1 to 3.0 for a prescribed value of $H/a$. This observation, which is more pronounced for larger values of $H/a$ (i.e. tall slender buildings), has also been reported in previous studies (e.g. [22, 23]). On the other hand, non-vertical SH waves tend to reduce the response for higher frequencies (e.g. $\omega/\omega_x > 1.0$), especially for smaller values of $H/a$ (e.g. $H/a < 1.0$) and larger values of $a_x$ (e.g. $a_x > 1.5$). This reduction in response is more pronounced with increasing $c_{app}$. It is also worth mentioning that the amplitude of the first peak response is generally insensitive to the variation of $c_{app}$. Finally, the variation of $H/a$ appears to have negligible effects on the response when $a_x$ is very small (e.g. $a_x = 0.1$). Note that a very small value of $a_x$ implies that the soil is much stiffer than the superstructure, and thus the rocking response of the foundation is expected to be small. Therefore, the slenderness ratio $H/a$, which is associated with the response of the superstructure due to foundation rocking, is not important for small $a_x$ values.

### 4.2. Translational response

In this section, a detailed analysis is performed to examine the translational response of all 40 buildings considered in this study under the action of simplified near-fault pulse-like ground motions. The seismic excitation input is described using the mathematical model proposed by Mavroeidis and Papageorgiou [12] (see Section 2). The translational displacement at the top of the superstructure (i.e. termed ‘translational displacement’ for simplicity) is first calculated in the frequency domain by multiplying the transfer function $U_H(\omega)$ of Equation (20) by the Fourier transform $D(\omega)$ of the simplified ground displacement (see Section 2). The translational response in the time domain is then obtained by applying the inverse fast Fourier transform. The response quantity of interest is the maximum absolute value of the translational displacement time history.

![Figure 2](https://example.com/figure2.png)
4.2.1. Effect of ground motion parameters. Three buildings with short, intermediate, and long periods are selected to examine the effect of ground motion parameters on translational response: Building A10 (3 stories, \(H = 14.4\) m, \(a = 22.9\) m, \(T_x = 0.28\) s and \(\beta = 234.1\) m/s), Building A36 (6 stories, \(H = 23.1\) m, \(a = 15.2\) m, \(T_x = 1.07\) s and \(\beta = 362.1\) m/s), and Building A28 (19 stories, \(H = 95.8\) m, \(a = 28.0\) m, \(T_x = 3.45\) s and \(\beta = 298.7\) m/s). Figure 3 presents the maximum translational displacement of the three buildings as a function of \(T_P/T_x\) for different values of \(\gamma\), whereas \(\beta/c_{\text{app}}\) is considered to be 0.1. Because the \(T_P\) values considered in this study range from 0.5 to 10 s (corresponding to \(M_W 5.2-7.8\)), the effective range of \(T_P/T_x\) over which Figure 3 is plotted differs for each of the three buildings.

The peak of the maximum translational displacement is attained when \(T_P/T_x\) approaches a value slightly larger than one (see Buildings A36 and A28), indicating that resonance occurs when the pulse period approaches the fundamental translational period of the building. This observation is consistent with results reported previously regarding the resonance response of single-degree-of-freedom (SDOF) systems subjected to near-fault pulse-like ground motions (e.g. [24, 25]). In addition, as \(\gamma\) increases from 1 to 3, the peak of the maximum translational displacement increases by a factor of \(\approx 2\), and the shape of the response curve changes gradually from ‘flattened’ to ‘peaked’. On the other hand, Building A10 is not subjected to resonance due to the range of \(T_P/T_x\) values considered in this study, thus resulting in significantly smaller response. Note that for generating Figure 3, parameter \(\beta/c_{\text{app}}\) is set equal to 0.1. Additional simulations for \(\beta/c_{\text{app}} = 0.2, 0.3, 0.4, 0.5\) reveal that \(\beta/c_{\text{app}}\) exerts an insignificant effect on the maximum translational displacement. This can be attributed to the fact that buildings, under the action of simplified long-period pulses, vibrate primarily in their fundamental mode, and the amplitude of the transfer function at the fundamental mode is generally insensitive to \(\beta/c_{\text{app}}\) (see Section 4.1.3 and Figure 2).

To further demonstrate the dominant effect of the fundamental mode, a similar parametric analysis was conducted for all 40 buildings with the response quantity of interest now being the overall maximum translational displacement between any point along the height of the building and its base. The analysis results (not shown here) suggested that the maximum translational displacement occurs at the top of the building for the vast majority of cases. It is only for a few long-period buildings (e.g. \(T_x > 3.0\) s) with \(T_P/T_x < 0.4\) that the maximum displacement at the top may slightly underestimate the overall maximum displacement along the height of the building due to higher mode effects.

4.2.2. Effect of shear wave velocity. In order to investigate the effect of shear wave velocity on translational response, four representative values of \(\beta\) are selected, corresponding to the National Earthquake Hazards Reduction Program Site Classes B, C, D, and E [26]. Figure 4a presents the
maximum translational displacement of the three buildings shown in Figure 3 as a function of $T_p/T_x$ for $\beta = 170, 270, 560, 1000 \text{ m/s}$, $\gamma = 2.0$ and $\beta/c_{\text{app}} = 0.1$. It appears that the maximum translational displacements of Buildings A10 and A36 are insensitive to $\beta$, whereas the response curve for Building A28 is only slightly affected by $\beta$.

Figure 4b presents parametric analysis results for three additional buildings exhibiting more noticeable effects of $\beta$: Building A30 (6 stories, $H = 34.0 \text{ m}$, $a = 57.6 \text{ m}$, $T_x = 0.92 \text{ s}$), Building B12 (32 stories, $H = 130.6 \text{ m}$, $a = 28.3 \text{ m}$, $T_x = 1.84 \text{ s}$), and Building B3 (47 stories, $H = 180.3 \text{ m}$, $a = 26.2 \text{ m}$, $T_x = 5.03 \text{ s}$). For Buildings A30, B12, and B3, as $\beta$ decreases from 1000 to 170 m/s, the peak response decreases, and the corresponding value of $T_p/T_x$ shifts towards larger values. This is consistent with findings reported by Veletsos and Meek [23] for building–foundation systems subjected to pulse-type excitations. This observation, which is not always evident in the response curves of buildings (e.g. Building A36 in Figure 4a), requires an explanation. A decrease in $\beta$ translates into an increase in $a_x = a/\omega_0$, which in turn implies that the amplitude of the first peak response of the transfer function and the corresponding value of $\omega_0$ both decrease (see Section 4.1.3 and Figure 2). These effects on transfer function tend to decrease the peak value of the maximum translational response and increase the corresponding value of $T_p/T_x$ as shown in Figure 4b. However, the reduction in the amplitude of the peak response also depends on the corresponding value of $T_p$, which affects the Fourier transform $D(\omega)$ of the seismic excitation input [12, 11]. In comparison with Buildings A10, A36, and A28 (Figure 4a), the more pronounced effect of $\beta$ on the response of Buildings A30, B12, and B3 (Figure 4b) is attributed to the particular values of $H/a$ and $a_x$, which directly affects the respective transfer functions (Figure 2). Finally, it should be noted that additional simulations for $\beta/c_{\text{app}} = 0.2, 0.3, 0.4, 0.5$ reveal that parameter $\beta/c_{\text{app}}$ has a

Figure 4. (a) Maximum translational displacement at the top of Buildings A10, A36, and A28 vs. $T_p/T_x$ for $\beta = 170, 270, 560, 1000 \text{ m/s}$, $\gamma = 2.0$ and $\beta/c_{\text{app}} = 0.1$. (b) Same parametric analysis for Buildings A30, B12, and B3. The dotted vertical line specifies the lower limit of the effective range of $T_p/T_x$ for each building.

negligible effect on the analysis results shown in Figure 4 because of the dominance of the fundamental mode response.

4.2.3. Effect of building parameters. Figure 5 shows the response curves of all 40 buildings as a function of $T_P/T_x$ and $T_x$ for $\beta = 170$, 270, 560, 1000 m/s, $\gamma = 2.0$, and $\beta/c_{app} = 0.1$. Inspection of the figure reveals that all response curves have a similar shape and the peak value of the maximum translational displacement occurs at $T_P/T_x \approx 1.1$. This observation is in agreement with findings reported previously for elastic SDOF systems subjected to near-fault pulse-like ground motions (e.g. [24, 25]). The similarity between results obtained from the shear beam model and the SDOF system can be attributed to the fact that the shear beam model, under the action of simplified long-period pulses, vibrates primarily in its fundamental mode. Figure 5 also illustrates that the maximum translational displacement for a fixed $T_P/T_x$ value tends to increase with increasing values of $T_x$. It is worth mentioning that, for small values of $\beta$ (i.e. 170 and 270 m/s) and depending on the values of $H/a$ and $a_x$, the peak displacement response of certain buildings may occur at $T_P/T_x$ values greater than 1.1 due to the effect of shear wave velocity on maximum translational displacement (see Section 4.2.2 and Figure 4). Finally, it should be noted that the variation of $\beta/c_{app}$ from 0.1 to 0.5 has an insignificant effect on the response curves of all 40 buildings shown in Figure 5.

Based on the parametric analysis results presented in Sections 4.2.1–4.2.3, it appears that the translational response of buildings to near-fault pulse-like ground motions is primarily affected by $\gamma$, $T_P$, and $T_x$, whereas $\beta$ may also have measurable effects depending on parameters $a_x$ and $H/a$.

5. TORSIONAL RESPONSE DUE TO SIMPLIFIED NEAR-FAULT PULSE-LIKE GROUND MOTIONS

The torsional response in the frequency domain of the soil–foundation–structure system shown in Figure 1a subjected to an obliquely incident plane SH wave has been derived analytically by Luco [3]. For the sake of brevity, only the final solution is presented in this section.

![Figure 5](image)

Figure 5. Maximum translational displacement at the top of all 40 buildings vs. $T_P/T_x$ and $T_x$ for $\beta = 170$, 270, 560, 1000 m/s, $\gamma = 2.0$ and $\beta/c_{app} = 0.1$. 

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The base twist $a_b$ of the superstructure (see Figure 1c) is provided by

$$a_b(\omega) = \frac{S\left(a_0 \frac{\omega}{\omega_0}, \Theta\right)}{1 - \frac{\pi}{20} \left(\frac{a_0 \omega}{\omega_0}\right)^2 \frac{\ln(t_1 + t_f)}{K_{TT}(0)}} K_{TT}(a_0 \frac{\omega}{\omega_0})$$

where

$$S\left(a_0 \frac{\omega}{\omega_0}, \Theta\right) = \alpha^x / u_{g0}, \quad a_0 = a_0/\beta = a_0/\omega_0, \quad k_{bd} = \frac{\pi}{2} \left(\frac{\omega_0}{\omega_0}\right) \sqrt{1 + 2\zeta_0^2}$$

$$I_s = \frac{4\pi}{15} \rho_s a^5, \quad I_b = \frac{1}{2} m_b a^2, \quad I_0 = \frac{1}{2} m_0 a^2$$

$$\alpha^x = \left(8iGa^3 \cos \theta_0 \int_0^t \theta_s(t) dt / K_{TT}(a_0)\right) u_{g0} / a, \quad K_{TT}(a_0) = 16Ga^3 \int_0^t \theta_R(t) dt$$

Note that $\omega_0$ is the fundamental torsional frequency, $\zeta_0$ is the hysteretic damping factor for torsional vibrations, $a_0$ is the relative stiffness parameter for torsional vibrations, and $\theta_R$ and $\theta_S$ are functions associated with the solution of the Fredholm integral equations.

Once the base twist $a_b$ has been obtained from Equation (22), the top twist $a_t$ (see Figure 1c) is calculated by

$$a_t(\omega) = a_t(\omega) \sec(k_{bd} H)$$

and the relative twist $\alpha_r$ between the top and the base of the superstructure is defined as

$$\alpha_r(\omega) = a_t(\omega) - a_b(\omega)$$

In summary, the transfer function of the relative twist of the superstructure expressed by Equation (25) is described in terms of the following dimensionless parameters: normalized frequency $\omega_0/\omega_0$, relative stiffness parameter $a_0$ for torsional vibrations, ratios of moments of inertia $I_s/I_b$ and $I_0/I_s$, hysteretic damping factor $\zeta_0$ for torsional vibrations, and angle of incidence $\Theta$.

The uncoupled torsional-to-translational frequency ratio is denoted by $\Omega = \omega_0/\omega_0 = T_\theta/T_x$, with $\Omega \geq 1$ implying a torsionally stiff system and $\Omega < 1$ implying a torsionally flexible system. A torsionally stiff system normally has stiffer members located towards the periphery of the structure, whereas a torsionally flexible system has stiffer members located towards its center. Several studies in the published literature have recommended representative values of $\Omega$. For instance, Chandler and Hutchinson [27] suggested that most buildings have a frequency ratio $\Omega$ in the range of 0.5−1.5 and values below 0.5 are very unlikely to occur. Based on analysis results for an idealized one-storey building consisting of a rigid deck supported on frames or walls, Riddell and Santa-Maria [28] concluded that the maximum feasible value of $\Omega$ for a circular slab is equal to 2. In addition, full-scale data from artificial and natural vibrations have indicated that the frequency ratio $\Omega$ may vary between 0.8 and 2.0 with a mean value of ~1.3 (e.g. [29, 30]).

Because no information is available about the fundamental torsional period $T_0$ of the 40 buildings, the frequency ratio $\Omega$ is introduced as a free parameter to determine $T_0$ from $T_x$. For the parametric analysis presented in this study, the frequency ratio $\Omega$ is assumed to vary between 0.5 and 2.0, yet one should keep in mind that torsionally stiff systems ($\Omega \geq 1$) are more common in design practice. Once the frequency ratio $\Omega$ is selected, the fundamental torsional period $T_0$ and the relative stiffness parameter $a_0$ for torsional vibrations may be calculated for each building using the fundamental translational period $T_x$. In addition, the ratio of moments of inertia $I_0/I_s$ can be determined by combining Equations (21) and (23d−e) resulting in $I_0/I_s = 1.875(\rho_s/\rho_0)(H/\Delta)$, whereas parameter $I_0/I_s$ may be obtained by multiplying $I_0/I_s$ by the foundation-to-superstructure mass ratio $\delta_m$. Subsequently, using the values of $\rho_s$, $\rho_0$, and $\delta_m$ specified in Section 4.1.2, $I_0/I_s$ and $I_0/I_s$ can readily
be calculated for all buildings. The hysteretic damping ratios for torsional and translational vibrations are assumed to be equal (i.e., $\zeta_0 = \zeta_x = 4\%$). Finally, the angle of incidence $\Theta$ is determined by considering typical values of the wave apparent velocity $c_{\text{app}}$ and the shear wave velocity $\beta$ (i.e., $\cos \Theta = \beta / c_{\text{app}}$). Using empirical and numerical approaches, previous studies have suggested that $c_{\text{app}}$ values of body waves in the near-fault region range between 2.0 and 3.8 km/s [4, 31, 32]. Moreover, the torsional response due to wave passage effects in the near-fault region has been found to increase for smaller values of $c_{\text{app}}$ [4, 11]. In this study, the wave apparent velocity $c_{\text{app}}$ is set equal to 2.0 km/s to obtain the most critical torsional response.

The interested reader is referred to Cao et al. [11] for a comprehensive parametric analysis of the transfer function of the relative twist expressed by Equation (25) and of the torsional response of the soil–foundation–structure system subjected to near-fault pulse-like ground motions. The present study further evaluates the increase in building displacements due to the torsional response arising from wave passage effects by means of accidental eccentricity.

6. EVALUATION OF ACCIDENTAL ECCENTRICITY

According to De la Llera and Chopra [7], the design accidental eccentricity arising from base rotations may be determined in such a way that, when used in conjunction with the equivalent static lateral forces, the increase in building displacement is identical to that computed from response history analysis. Based on analysis results from idealized single-story systems, De la Llera and Chopra [7] derived mathematical expressions for the accidental eccentricity of symmetric- and asymmetric-plan buildings. Because the soil–foundation–structure system considered in this study primarily vibrates in its fundamental mode, the formulation proposed in [7] is adopted to estimate the accidental eccentricity arising from base torsional motions due to wave passage effects.

Assuming that $u_x(t)$ denotes the translational displacement at the top of the superstructure and $u_y(t)$ represents the relative twist between the top and the base of the superstructure, the normalized response at $y = \pm r$ is defined as

$$
(u_{x\pm})(0) = \frac{\max|u_x(t)\pm ru_y(t)|}{\max|u_x(t)|}
$$

(26)

where $r$ is the radius of gyration of the system plan about a vertical axis passing through the center of mass. For the symmetric cylindrical model shown in Figure 1, this vertical axis coincides with the $z$ axis, and the radius of gyration is equal to $a / \sqrt{2}$. Note that quantity $ru_y(t)$ may be interpreted as the tangential displacement calculated at distance $r$ along the $y$ axis arising from base torsional motions due to wave passage effects. The normalized response $u_0$, defined as the larger value of $(\bar{u}_+)_{0}$ and $(\bar{u}_-)_{0}$, provides a measure of the amplification in translational response due to the presence of torsional vibrations. For symmetric buildings with a circular floor plan, the normalized accidental eccentricity $e_a / 2a$ is provided by

$$
e_a / 2a = \frac{\sqrt{2}}{4}(u_0 - 1) \Theta^2
$$

(27)

6.1. Parametric analysis

Following the selection of input parameters presented in Sections 2, 4, and 5, the translational and torsional components of the building response due to near-fault pulse-like motions are calculated for all 40 buildings. Four characteristic values of $\beta$ (i.e., 170, 270, 560, 1000 m/s) are considered along with a fixed value of $c_{\text{app}} = 2.0$ km/s (see Section 5), resulting in $\beta / c_{\text{app}} = 0.085, 0.135, 0.280, 0.500$. The frequency-dependent function $S_{\text{xx}}$ is evaluated by interpolating the tabular data provided by Luco and Mita [19] at the considered values of $\beta / c_{\text{app}}$. Once the translational and torsional components of the building response have been computed, the normalized response and the normalized accidental eccentricity are evaluated using Equations (26) and (27).
6.1.1. Effect of frequency ratio and pulse period. Figure 6a and b shows, respectively, the maximum translational displacement (max\(u(t)\)) and the maximum displacement due to torsional vibrations (max|\(ru(t)\)|) as a function of \(\Omega\) and \(T_p/T_x\) for Buildings A10, A36, and A28. Note that max\(u(t)\) is significantly larger than max|\(ru(t)\)| (even for Building A10), implying that the normalized response \(\bar{u}_0\) occurs at a time instant \(t'\) that coincides, for all practical purposes, with the time of max\(u(t)\). Figure 6c illustrates the displacement amplitude due to torsional vibrations at \(t_0\) (|\(ru(t_0)\)|), whereas Figure 6d and e presents the normalized response \(e_u/2a\) and the normalized accidental eccentricity \(e_a/2a\). In generating Figure 6, it was assumed that \(\beta = 270\) m/s and \(\gamma = 2.0\).

- For Building A10 (\(T_x = 0.28\) s), a value of \(\Omega = 0.5\) implies that \(T_\theta = 0.56\) s, which is close to the lower bound of the \(T_p\) values considered in this study (i.e. 0.5–10 s). As a consequence, resonance occurs only in torsional response and large values of max|\(ru(t)\)| are observed. In addition, although the peak value of \(\bar{u}_0\) for Building A10 is the largest among the three buildings, its peak value of \(e_a/2a\) is the smallest. This may be interpreted by the fact that \(e_a/2a\) incorporates the combined effects of \(e_u\) and \(\Omega^2\) as indicated by Equation (27).

- For Building A36 (\(T_x = 1.07\) s), the variation of \(\Omega\) from 0.5 to 2.0 results in \(T_\theta\) ranging from 2.14 to 0.54 s, which in turn affects the values of \(T_p/T_x\) corresponding to torsional resonance. It should also be noted that, with the exception of \(\Omega \approx 1.0\), large values of |\(ru(t_0)\)| and \(\bar{u}_0\) are observed when torsional resonance occurs. When \(\Omega \approx 1.0\), |\(ru(t_0)\)| is very small, and \(\bar{u}_0\) approaches unity, implying that torsional response has a negligible contribution to the increase in building displacements. This trend is schematically explained in Figure 7, where Building A36 with \(\Omega = 1.0\) (\(T_x = T_\theta = 1.07\) s) is excited by a simplified ground motion pulse. Without loss of generality, let \(T_p = 1.2\) s (i.e. to induce torsional resonance), \(\gamma = 2.0\) and \(\beta = 270\) m/s. Figure 7a compares the phase angles of the transfer functions of translational displacement (\(U_H/u_g(0)\)) and displacement due to torsional vibrations (\(r_\alpha r/u_g(0)\)). Note that the phase of \(U_H/u_g(0)\) has been shifted by 90°, yielding a variation of phase angle with normalized frequency that is
almost identical to that of \( \nu_0 \). This implies that the torsional response due to wave passage effects is 90° out of phase with respect to the translational response. Figure 7b shows the time histories of \( \nu_0(t) \) and \( \nu_0(t') \), with the latter scaled up by a factor of 40 to indicate the waveforms more clearly. It is evident that the two time signals are 90° out of phase, and therefore when \( \nu_0 \) attains its peak value, \( \nu_0 \) is close to zero, thus resulting in values of \( \nu_0 \) close to unity. The negligible contribution of the torsional response to the increase in building displacements for buildings with \( \Omega \approx 1.0 \) has previously been noted by De la Llera and Chopra [7, 33, 34] and is further confirmed in this study.

- For Building A28 (\( T_x = 3.45 \) s), similar trends can be observed for the variation of max\( |\nu_0(t)| \) and \( |\nu_0(t')| \) with respect to \( \Omega \) and \( T_P/T_x \). However, it is interesting to note that, although local peaks of \( \nu_0 \) are observed when torsional resonance takes place, the global peak of \( \nu_0 \) occurs at \( T_P/T_x = 0.14 \) (i.e. \( T_P = 0.5 \) s). This observation can be attributed to higher mode effects. Cao et al. [11] reported local amplification of maximum torsional response for small \( T_P/T_x \) values due to the excitation of higher modes, whereas the parametric analysis presented in Section 4 indicated that higher mode effects are relatively insignificant for the maximum translational response. As a consequence, an increased value of \( \nu_0 \) may be obtained, when higher modes are excited.

6.1.2. Effect of parameter \( \gamma \). Figure 8 shows the normalized accidental eccentricity \( e_\gamma/2a \) for Buildings A10, A36, and A28 as a function of \( \Omega \) and \( T_P/T_x \) for \( \gamma = 1.0, 1.5, 2.0, 3.0 \) and \( \beta = 270 \) m/s. The variation of \( \gamma \) has a significant effect on the Fourier amplitude spectrum of the ground displacement pulses [12, 11], which in turn affects both the translational and torsional components of the structural response. For Buildings A36 and A28, as \( \gamma \) increases from 1 to 3, the peak values of \( e_\gamma/2a \) increase from 0.05 to 0.07 and from 0.03 to 0.08, respectively. However, this trend does not necessarily apply to all buildings (e.g. Building A10 in Figure 8; see also Figure 11). In fact, an increase in \( \gamma \) may result in an increase or decrease in the peak value of \( e_\gamma/2a \) depending on the building properties.

6.1.3. Effect of shear wave velocity. For Buildings A10, A36, and A28, the variation of \( \beta \) has negligible effects on \( e_\gamma/2a \). This observation can be attributed to the following factors: (i) the torsional response has been found to be controlled by the wave apparent velocity rather than the shear wave velocity [11], and (ii) the maximum translational displacement of these three buildings is generally insensitive to \( \beta \) as shown in Figure 4a. On the other hand, the effects of \( \beta \) on \( e_\gamma/2a \) are more noticeable for the three buildings shown in Figure 4b. Figure 9 presents the variation of \( e_\gamma/2a \) for Buildings A30, B12, and B3 as a function of \( \Omega \) and \( T_P/T_x \) for \( \beta = 170, 270, 560, 1000 \) m/s and \( \gamma = 2.0 \). For a particular building and as \( \beta \) varies, the contours of \( e_\gamma/2a \) are of similar shape, indicating that \( \beta \) contributes to the variability of \( e_\gamma/2a \) to a lesser extent than other parameters (e.g. \( \Omega, T_P \) and \( \gamma \)). As \( \beta \) increases, the peak value of \( e_\gamma/2a \) increases from 0.15 to 0.21 for Building A30, whereas it decreases from 0.12 to 0.07 for Building B12 and from 0.03 to 0.02 for Building B3.
6.2. Comparison with typical code-prescribed values of accidental eccentricity

Figure 10 shows the variation of \(\frac{e_a}{2a}\) for 15 representative buildings as a function of \(\Omega\) and \(\frac{T_P}{T_x}\) for \(\gamma = 1.0, 1.5, 2.0, 3.0\) and \(\beta = 270\) m/s. Note that the lower limit of \(\frac{T_P}{T_x}\) differs for each building (see Figure 3), whereas the upper limit of \(\frac{T_P}{T_x}\) and the colorbar range are adjusted to emphasize the trends. [Colour figure can be viewed at wileyonlinelibrary.com]

Among all 40 buildings, the peak value of \(\frac{e_a}{2a}\) exceeds 10% for three buildings and 5% for 11 more buildings.

In generating Figure 10, parameters \(\beta\) and \(\gamma\) were set to fixed values, and therefore the results shown in the figure do not reflect the effect of these two parameters. The shear wave velocity was previously found to have only a limited influence on \(\frac{e_a}{2a}\) (see Section 6.1.3). On the other hand, \(\gamma\) affects both the translational and torsional responses and may have an important effect on \(\frac{e_a}{2a}\) (see Section 6.1.2). Figure 11a presents the peak values of \(\frac{e_a}{2a}\) for all 40 buildings as a function of the foundation radius for \(\gamma = 1.0, 1.5, 2.0, 3.0, \beta = 270\) m/s, and \(\Omega = 0.5\)–2.0. Figure 11a suggests that (i) the variation of \(\gamma\) may have a significant effect on \(\frac{e_a}{2a}\), but no general trend can be observed, and (ii) there exists a strong tendency of increasing \(\frac{e_a}{2a}\) with increasing foundation radius [7, 10]. By taking into consideration the variation of \(\gamma\), the peak value of \(\frac{e_a}{2a}\) exceeds 10% for seven buildings and 5% for 19 more buildings.
Figure 9. Normalized accidental eccentricity $e_i/2a$ for Buildings A30, B12 and B3 vs. $\Omega$ and $T_p/T_x$ for $\beta = 170, 270, 560, 1000$ m/s and $\gamma = 2.0$. Note that the lower limit of $T_p/T_x$ differs for each building (see Figure 4b), whereas the colorbar range is adjusted to emphasize the trends. [Colour figure can be viewed at wileyonlinelibrary.com]

Figure 10. Normalized accidental eccentricity $e_i/2a$ for 15 representative buildings vs. $\Omega$ and $T_p/T_x$ for $\beta = 270$ m/s and $\gamma = 2.0$. Buildings are displayed in order of increasing $T_x$ (from left to right in rows). Values within the parentheses denote the fundamental translational period and the foundation radius of each building. Note that the lower limit of $T_p/T_x$ differs for each building, whereas the upper limit of $T_p/T_x$ is adjusted to emphasize the trends. [Colour figure can be viewed at wileyonlinelibrary.com]
Note that the peak values of $e_{a/2}$ shown in Figure 11a were computed for $\Omega$ varying between 0.5 and 2.0, thus implying that the majority of buildings attain their peak $e_{a/2}$ values at either $\Omega = 0.5$ or $\Omega = 2.0$ (according to Figure 10 and the preceding discussion). It may then be argued that the results shown in Figure 11a are not necessarily representative of the most typical design practices of actual buildings because these two extreme values of $\Omega$ are relatively uncommon. Following the discussion on the practical range of $\Omega$ in Section 5, a value of $\Omega = 1.25$ (i.e. $T_{0} = 0.8T_{x}$) is selected to represent the frequency ratio of realistic buildings. Figure 11b presents the same analysis as in Figure 11a with the exception that $\Omega$ is fixed to 1.25. Under this condition, the observations made with regard to Figure 11a still hold true, but the peak values of $e_{a/2}$ are significantly smaller. As a matter of fact, it is only for the three buildings (A17, A24, and A30) with the largest foundation radius that the peak value of $e_{a/2}$ exceeds 5% for certain values of $\gamma$.

6.3. Effect of broadband ground motions

Mavroeidis and Papageorgiou [12] proposed a simplified methodology for generating broadband near-fault pulse-like ground motions adequate for engineering analysis and design. According to this methodology, the coherent (long-period) ground motion component is simulated using the mathematical model presented in Section 2, whereas the incoherent (high-frequency) seismic radiation is synthesized using the specific barrier model [35–37]. Using this approach, Cao et al. [11] simulated time histories of broadband ground motions in the near-fault region for three hypothetical earthquakes of $M_W$ 5.8, 6.4, and 7.0, representing moderate, moderate-to-large, and large seismic events, respectively. These synthetic motions are used in this section to assess the effectiveness of idealized pulse models in estimating the normalized accidental eccentricity.

Figure 12 illustrates the normalized accidental eccentricity $e_{a/2}$ for all 40 buildings as a function of $T_{x}$ for $\Omega = 1.25$ due to the three hypothetical seismic events. For the $M_W$ 5.8, 6.4, and 7.0 earthquakes, the agreement between the values of $e_{a/2}$ due to the long-period and broadband ground motions is very good for $T_{x} > 0.5$ s, 1.0 s and 2.0 s, respectively. For shorter periods, the normalized accidental eccentricity due to the broadband ground motion is clearly underestimated when only the long-period ground motion component is used in the analysis. This observation may be explained by the fact that the seismic response of shorter period buildings is primarily affected by the high-frequency component of the broadband ground motion, implying that the long-period component by itself is not capable of capturing the translational and torsional responses, thus resulting in the observed differences between the computed $e_{a/2}$ values.

Figure 12 also shows that short-period buildings ($T_{x} < 1.0$ s) subjected to broadband ground motions, in general, tend to have larger $e_{a/2}$ values than long-period buildings ($T_{x} > 1.0$ s), especially for the $M_W$ 6.4 and 7.0 earthquakes. In addition, the normalized accidental eccentricities for all 40 buildings subjected to broadband ground motions are less than the typical code-prescribed...
value of 5%. It is only for Building A17 (circled by dashed line in Figure 12) that $e_a/2a$ reaches values of 6 and 11% for $M_w$ 6.4 and 7.0, respectively. This can be attributed to the 75-m foundation radius of Building A17, which is the largest among all 40 buildings.

7. CONCLUSIONS

This study has investigated the characteristics of the accidental eccentricity in symmetric buildings due to torsional response arising from wave passage effects in the near-fault region. The main contributions and findings of this study are summarized as follows:

- The transfer functions of the translational and rocking response of the foundation and of the translational response of the superstructure were derived analytically assuming that the superstructure behaves as a shear beam under the action of translational and rocking base excitations arising from the propagation of an obliquely incident plane SH wave. Parametric analysis results indicated that the modulus of translational response at the top of the superstructure is primarily sensitive to the slenderness ratio $H/a$ and the relative stiffness parameter $a_\alpha$ for translational vibrations.
- The overall maximum translational displacement due to near-fault pulse-like ground motions is attained when the pulse period $T_p$ of the ground excitation approaches the fundamental translational period $T_x$ of the building and is increased by a factor of ~2 as parameter $\gamma$ increases from 1 to 3. The shear wave velocity $\beta$ may also have measurable effects on the overall maximum translational displacement depending on parameters $a_\alpha$ and $H/a$.
- The normalized accidental eccentricity is most sensitive to the pulse period $T_p$ of the near-fault ground motions and to the uncoupled torsional-to-translational fundamental frequency ratio $\Omega$ of the structure. Parameter $\gamma$ may also have a significant effect on the normalized accidental eccentricity, whereas the shear wave velocity $\beta$ affects the normalized accidental eccentricity to a lesser extent. It was also found that buildings with larger foundation radius exhibit, in general, larger normalized accidental eccentricity.
- The normalized accidental eccentricity due to broadband near-fault ground motions is well approximated by simplified pulse-like ground motions for longer period buildings, while it is underestimated for shorter period buildings. For symmetric buildings with values of $\Omega$ commonly used in design practice, the normalized accidental eccentricity due to wave passage effects is less than the typical code-prescribed value of 5%, except for buildings with very large foundation radius.

It should be noted that these findings were obtained for the soil–foundation–structure system considered in this study, i.e., superstructure modeled as a shear beam and foundation represented by a rigid circular disc. Tall slender buildings may experience significant bending, and therefore a model of the superstructure that takes into account both shear and bending deformations should be considered. In addition, further investigation should be carried out considering elongated rectangular
foundations, for which the wave passage effects are expected to be more pronounced. It should also be mentioned that the accidental eccentricity prescribed by building codes is intended to encompass the effects of accidental torsion induced not only from wave passage, but also from ground motion incoherence and uncertainties in the distributions of stiffness and mass. Moreover, nonlinear soil response, ground failure, and soil liquefaction may induce additional transient and permanent rotations on the ground surface near faults [38], which in turn may affect torsional response. Therefore, torsional effects arising from different sources need to be considered to assess the adequacy of code-prescribed values of accidental eccentricity to account for accidental torsion in symmetric buildings subjected to near-fault pulse-like ground motions.

NOMENCLATURE

\( A, \gamma, \nu, f_p, T_p, t_0, V(\omega), D(\omega) \) Near-fault pulse parameters (amplitude, number of cycles, phase, prevailing frequency, pulse period, time shift)

\( H, A, \rho_n \) Height, floor area, and mass density of superstructure

\( a \) Foundation radius

\( m_s, m_0 \) Masses of superstructure and foundation

\( I_n, I_0 \) Mass moments of inertia of superstructure and foundation about \( z \) axis

\( J_0 \) Mass moment of inertia of foundation about \( y \) axis

\( \omega_x, \omega_0 \) Uncoupled fixed-base fundamental translational and torsional frequencies

\( \zeta_x, \zeta_0 \) Hysteretic damping factors for translational and torsional vibrations

\( \beta, G, \sigma, \rho_s \) Soil properties (shear wave velocity, shear modulus, Poisson’s ratio, density)

\( \Theta \) Angle of incidence

\( u_{g0} \) Amplitude of free-field motion

\( U^{rx}, U^{sx} \) Motion of incident wave

\( U_{g0}, \Psi_0 \) Transfer functions of translational and rocking response of foundation

\( U(z), U_0, \Psi_0 \) Transfer function of translational response of superstructure at level \( z \)

\( U_{g0}, \Psi_0 \) Transfer function of translational and rocking components of foundation input motion

\( U_r, \Psi_r \) Motions of foundation associated with soil deformation

\( S_{ss} \) Frequency-dependent function

\( a_0 \) Dimensionless frequency

\( c_{app} \) Wave apparent velocity

\( V_s, M_s \) Force and moment that foundation exerts on soil

\( K(\omega) \) Frequency-dependent impedance matrix

\( K_{HH}, K_{HM}, K_{MH}, K_{MM} \) Normalized horizontal, coupling, and rocking impedance functions

\( k \) Shear stiffness

\( m \) Mass per unit length

\( V(z) \) Shear force at level \( z \)

\( V_s, M_s \) Base shear and base moment

\( a_x, a_0 \) Relative stiffness parameters for translational and torsional vibrations

\( m_s, J_s, I_s \) Soil mass and soil moments of inertia

\( U_H \) Transfer function of translational response at top of superstructure

\( H_l a \) Slenderness ratio

\( T_x, T_0 \) Fixed-base fundamental translational and torsional periods

\( \delta_m \) Foundation-to-superstructure mass ratio

\( \alpha \) Foundation input twist

\( a_0, a_x, a_r \) Transfer functions of base twist, top twist, and relative twist

\( \theta_R, \theta_S \) Functions associated with the solution of Fredholm integral equations

\( \Omega \) Uncoupled torsional-to-translational fundamental frequency ratio

\( u_x(t) \) Translational displacement at top of superstructure
\[ u_0(t) \] Relative twist between top and base of superstructure
\[ \langle u_{az} \rangle_0 \] Normalized response at \( y = \pm r \)
\[ \bar{u}_0 \] Normalized response defined as \( \max \{ \langle u_{az} \rangle_0 \} \)
\[ r \] Radius of gyration
\[ e_r/2a \] Normalized accidental eccentricity
\[ t' \] Time instant when \( \bar{u}_0 \) occurs

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**REFERENCES**