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A generic analytical solution for modelling pumping tests in wells intersecting fractures

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Abstract

The behaviour of transient flow due to pumping in fractured rocks has been studied for at least the past 80 years. Analytical solutions were proposed for solving the issue of a well intersecting and pumping from one vertical, horizontal or inclined fracture in homogeneous aquifers, but their domain of application—even if covering various fracture geometries—was restricted to isotropic or anisotropic aquifers, whose potential boundaries had to be parallel or orthogonal to the fracture direction. The issue thus remains unsolved for many field cases. For example, a well intersecting and pumping a fracture in a multilayer or a dual-porosity aquifer, where intersected fractures are not necessarily parallel or orthogonal to aquifer boundaries, where several fractures with various orientations intersect the well, or the effect of pumping not only in fractures, but also in the aquifer through the screened interval of the well.

Using a mathematical demonstration, we show that integrating the well-known Theis analytical solution (Theis, 1935) along the fracture axis is identical to the equally well-known analytical solution of Gringarten et al. (1974) for a uniform-flux fracture fully penetrating a homogeneous aquifer. This result implies that any existing line- or point-source solution can be used for implementing one or more discrete fractures that are intersected by the well. Several theoretical examples are presented and discussed: a single vertical fracture in a dual-porosity aquifer or in a multi-layer system (with a partially intersecting fracture); one and two inclined fractures in a leaky-aquifer system with pumping either only from the fracture(s), or also from the aquifer between fracture(s) in the screened interval of the well. For the cases

30 with several pumping sources, analytical solutions of flowrate contribution from each
31 individual source (fractures and well) are presented, and the drawdown behaviour according
32 to the length of the pumped screened interval of the well is discussed. Other advantages of
33 this proposed generic analytical solution are also given.

34 The application of this solution to field data should provide additional field information on
35 fracture geometry, as well as identifying the connectivity between the pumped fractures and
36 other aquifers.

37 To protect this original concept of a generic solution for modelling pumping tests in fractured
38 media, a patent application has been deposited on parts of this work (French National Institute
39 of Industrial Property).

40 Key words: analytical solution, pumping in discrete fractures, fractured rocks, pumping test

41

42 **1. Introduction**

43 Since the late 1930s (e.g., Strelsova, 1988), much work has been carried out to characterize
44 the transient flow of pumping tests carried out in naturally or artificially fractured aquifers
45 (e.g. Muskat, 1937; Warren and Root, 1963; Russell and Truitt, 1964; Bertrand et al., 1980;
46 Barker, 1988; Moench, 1984; Hamm and Bidaux, 1996; Jourde et al., 2002; Tiab, 2005; Delay
47 et al., 2007; Rafini and Larocque, 2012; Dewandel et al., 2014; Roques et al., 2016). This led
48 to the development of several analytical solutions for understanding the flow behaviour
49 created by a well intersecting and pumping one vertical, horizontal or inclined fracture with
50 infinite or finite hydraulic conductivity embedded in a homogeneous aquifer (Gringarten and
51 Ramey, 1973, 1974; Gringarten et al., 1974; Cinco-Ley et al., 1975, 1998, Thiery, 1980; see
52 also the PetroWiki website). These solutions were obtained by applying the Green's and
53 source functions and the Newman's product method (Newman, 1936; Gringarten and Ramey,
54 1974). However, their domain of application, though proposed for a variety of fracture
55 geometries, is restricted to isotropic or anisotropic infinite aquifers that may be limited in
56 space by no-flow or constant-head boundaries (Gringarten et al., 1974), leaving several
57 possibilities unsolved. For example, pumping a fracture in a multilayer or dual-porosity
58 aquifer, where the fracture is not necessarily parallel or orthogonal to the aquifer boundaries,
59 or a well intersecting and pumping fractures with various orientations, or the effect of

60 pumping from both fractures and also the aquifer in front of the screened interval of the well,
61 etc.

62 The aim of our research was to seek a new and alternative solution for computing drawdown
63 while pumping in one or several fractures, based on existing line or point source solutions.
64 We first demonstrate that the solutions developed for pumping in a vertical fracture can
65 directly be found by integrating the Theis analytical solution (Theis, 1935) along the fracture
66 axis. Then, by extension, line-source solutions for dual-porosity and multi-layer aquifers and a
67 point-source solution for a leaky aquifer are used when pumping discrete fractures in such
68 aquifers. Theoretical examples are given for pumping in: *i*) A fracture in a dual-porosity
69 aquifer; *ii*) A fracture in a multi-layer system that fully or partially intersects one of the
70 aquifer layers; *iii*) An inclined fracture in a homogeneous aquifer; *iv*) Two inclined fractures
71 in a leaky-aquifer system with pumping only in the fractures (i.e. the well is only screened in
72 front of the fractures); and *v*) The same as *iv*), but now pumping in both the fractures and the
73 aquifer through the well itself (screened interval). As in *iv*) and *v*), the flowrate contributions of
74 each pumped source (i.e., the two fractures and the screened interval of well) to the total
75 pumping rate vary over time, analytical solutions for evaluating their relative contributions are
76 also presented.

77 We do not suggest that the proposed solution should replace existing models used for
78 modelling drawdown in pumping tests performed in homogeneously fractured media (e.g.,
79 Barker, 1988; Moench, 1984; Hamm and Bidaux, 1996), or for pumping in fractures in
80 homogeneous aquifers (e.g., Gringarten et al, 1974; Thiéry, 1980). Rather, they are meant to
81 supplement existing models by providing additional hydrogeological information.

82

83 **2. Mathematical demonstration**

84 Here, we demonstrate that the well-known Theis analytical solution (1935), defined for a well
85 fully penetrating an isotropic aquifer and integrated along a fracture axis, is strictly identical
86 to the analytical solution of pumping in one vertical fracture proposed by Gringarten et al.
87 (1974), with uniform flux distribution along the fracture plane. In this conceptual aquifer
88 model (Fig. 1), the well intercepts the middle of a vertical fracture of length $2x_f$ and negligible
89 thickness, intersecting a homogeneous and infinite aquifer of transmissivity T and storage

90 coefficient S . Hereafter, we briefly show how we achieved this demonstration; more details
 91 can be found in the *Supplemental Materials*.

92 Assuming that the conductivity of the fracture can be considered infinite, and integrating the Theis
 93 well function along the fracture plane leads to the following equation:

$$94 \quad s_I(x, y, t) = \frac{1}{4\pi T} \int_{-x_f}^{+x_f} q(x) E_1 \left(\underbrace{\frac{(x - x_{obs})^2 + y_{obs}^2}{4Tt} S}_{\text{This well function}} \right) dx \quad \text{Eq.1}$$

95 This well function

96 where $q(x)$ is the rate of pumping per unit length of the fracture, t the time since starting the
 97 pumping, and E_1 the exponential integral (see Fig. 1 for parameters that are not defined in the
 98 text). Assuming that the pumping rate Q is uniformly distributed along the fracture, then $q(x)$
 99 takes the following form:

$$100 \quad Q = \int_{-x_f}^{+x_f} q(x) dx \Rightarrow q(x) = q = \frac{Q}{2x_f} \quad \text{Eq.2}$$

101 Using this statement and the following dimensionless variables: $t_D = \frac{Tt}{x_f^2 S}$; $x_D = \frac{x_{obs}}{x_f}$;

102 $y_D = \frac{y_{obs}}{x_f}$, Eq.1 becomes:

$$103 \quad s_I(x, y, t) = \frac{Q}{2\pi T} \frac{1}{4x_f} \int_{-x_f}^{+x_f} \left(\int_0^{t_D} \frac{e^{-\left[\left(\frac{x}{x_f} - x_D\right)^2 + y_D^2\right]/4\tau}}{\tau} d\tau \right) dx \quad \text{Eq.3}$$

104 According to the Fubini theorem (τ and x being independent variables), the order of the
 105 integration can be inverted, and as $e^{-y_D^2/4\tau}$ does not depend on x , Eq.3 can be rearranged as:

$$106 \quad s_I(x, y, t) = \frac{Q}{2\pi T} \frac{1}{x_f} \int_0^{t_D} e^{-y_D^2/4\tau} \underbrace{\left(\int_{-x_f}^{+x_f} \frac{e^{-\left(\frac{x}{x_f} - x_D\right)^2/4\tau}}{4\tau} dx \right)}_{(a)} d\tau \quad \text{Eq.4}$$

107

108 When changing variable $v = \left(\frac{x}{x_f} - x_D \right) / \sqrt{2\sqrt{\tau}}$, the term (a) can be separated into two terms
 109 related to the *Erf* function, such as:

$$110 \quad a = \int_{-x_f}^{+x_f} \frac{e^{-\left(\frac{x}{x_f} - x_D\right)^2 / 4\tau}}{4\tau} dx = \frac{x_f \cdot \sqrt{\pi}}{4\sqrt{\tau}} \left[-\operatorname{Erf}\left(\frac{(1+x_D)}{2\sqrt{\tau}}\right) + \operatorname{Erf}\left(\frac{(1-x_D)}{2\sqrt{\tau}}\right) \right] \quad \text{Eq.5}$$

111 Combining Eq.4 and Eq.5, we obtain:

$$112 \quad s_I(x, y, t) = \frac{Q}{2\pi T} \int_0^{t_D} e^{-\frac{y_D^2}{4\tau}} \left[\operatorname{Erf}\left(\frac{1-x_D}{2\sqrt{\tau}}\right) - \operatorname{Erf}\left(\frac{1+x_D}{2\sqrt{\tau}}\right) \right] \frac{\sqrt{\pi}}{4\sqrt{\tau}} d\tau \quad \text{Eq.6}$$

113 Eq.6 demonstrates that the Theis analytical solution, when integrated along the fracture axis,
 114 corresponds exactly to the Gringarten et al. (1974) analytical solution (their Eq.20) using
 115 Green's and source functions and the Newman's product method for a vertical fracture with
 116 an uniform flux distribution that fully penetrates the aquifer.

117 This implies that the integration of any line-source solution (i.e. a well function defined for a
 118 fully or partially penetrating well) along the fracture axis can be used for computing
 119 drawdown caused by pumping from a vertical fracture that partially or fully penetrates the
 120 aquifer. In addition, any point-source solution can be used for computing drawdown while
 121 pumping an inclined or a vertical fracture that is intersected by a well.

122

123 3. Theoretical examples

124 This section presents theoretical examples based on the integration of known analytical line-
 125 or point-source solutions along the fracture plane. They are based on equation Eq.1 from
 126 which the inner integral (Theis well function in Eq.1) is replaced by another well function,
 127 assuming that flux is uniformly distributed along the fracture (Eq.2). Dimensionless
 128 drawdown (s_D) and its logarithmic derivative (s_D') vs. dimensionless time with respect to
 129 fracture length (t_{Dxf} or t_{DLf} , see appendix A) were computed for creating Log-Log diagnostic
 130 plots and examining transient flow regimes (Bourdet et al., 1983; Deruyck et al., 1992;
 131 Renard et al., 2009; Rafini et al., 2017, etc.). As benchmarks, the solutions were compared to
 132 existing analytical solutions for pumping in a vertical and a horizontal fracture in a
 133 homogeneous aquifer (Gringarten et al., 1974; Thiéry, 1980).

134 3.1 Vertical fractures

135 These analytical solutions are found by integrating line-source solutions. Because the
 136 analytical integration of a given line-source solution along the fracture can be too difficult, we
 137 approximate it by first dividing the fracture length into smaller segments and then placing a
 138 line-source solution at each segment. Because of the linear properties of the diffusivity
 139 equation, we used the principle of superposition, summing up drawdowns induced by each
 140 segment to provide the total drawdown value due to pumping in the fracture. Then, this
 141 general formulation is used for integrating the solutions numerically:

$$142 \quad s_{\omega-f}(x, y, t, x_f, \lambda, \alpha, \dots) = \frac{Q}{4\pi T \cdot M} \sum_{i=1,2,3..}^M s_{\omega}(x_i, y, t, \lambda, \alpha, \dots) \quad \text{Eq.7}$$

143 where $s_{\omega}(x, y, t, \lambda, \alpha, \dots)$ is a known line-source solution (e.g. dual-porosity, dual-
 144 permeability or partially penetrating well solutions) and $s_{\omega-f}(x_i, y, t, x_f, \lambda, \alpha, \dots)$ is the solution
 145 for pumping from a vertical fracture with half-length x_f (Fig. 1) in an aquifer whose
 146 parameters are defined in the s_{ω} solution (λ, α, \dots). The fracture is along the x -axis, M is the
 147 number of segments, and x_i is the abscissa of the middle of each segment ($-x_f \leq x_i \leq +x_f$).

148 3.1.1. Pumping at the centre of a vertical fracture fully penetrating a dual-porosity 149 medium

150 Figure 2 provides an example of the use of Eq.7 with Moench's (1984) dual-porosity model (a
 151 model frequently used for interpreting pumping tests performed in homogeneously fractured
 152 media). Here, the implemented fracture is vertical and fully penetrates the dual-porosity
 153 aquifer (e.g. Fig. 1). Figure 2 shows how pumping in a fracture behaves for a set of
 154 interporosity flow coefficients λ (see caption for explanation of λ). As expected, when $\lambda=0$
 155 (i.e. the matrix hydraulic conductivity is nil corresponding to a single-porosity aquifer), the
 156 solution is equivalent to the Gringarten et al. (1974) analytical solution, with an error of 0.2%
 157 ($error = |s_G - s_{\omega-f}| / s_G$, s_G being Gringarten's solution; Eq.6). The difference between the
 158 theoretical solution and its numerical evaluation shows that discretization (division of the
 159 fracture length into smaller elements) errors are very small.

160 At the start of pumping, as expected, the flow is linear (half-unit slope of derivative curves)
 161 and corresponds to flow from the most permeable medium (i.e. the secondary porosity) to the
 162 vertical fracture. For intermediate pumping stages, the derivative curves have a classical 'U'

163 shape, characterizing the flow from primary (the block matrix) to secondary porosity of the
 164 dual-porosity aquifer. Then, for late pumping stages, the derivatives form a plateau that
 165 corresponds to radial flow from the dual-porosity aquifer to the vertical fracture ($s_D'=0.5$). In
 166 experimental data, this implies that drawdown values on a semi-logarithmic plot form a
 167 straight line from which the aquifer transmissivity can be deduced.

168 *3.1.2. Pumping at the centre of a vertical fracture partially penetrating the deepest layer* 169 *of a multi-layer aquifer*

170 Our second example (Fig. 3a) corresponds to a multilayer aquifer, where pumping from the
 171 deepest aquifer induces depletion in the upper one. This system is characterized by a lower
 172 aquifer of thickness B with transmissivity T and storage coefficient S , and an upper aquifer
 173 with transmissivity T_0 , and storage coefficient S_y . Both aquifers are separated by an aquitard
 174 of hydraulic conductivity k' and thickness B' . The vertical fracture is located in the deeper
 175 aquifer and is characterized by its location in the aquifer (z_f = vertical coordinate of the
 176 fracture centre), its height h_f and its length $2x_f$. The line-source solution for this conceptual
 177 model is an extension of the Hunt and Scott (2007) model for a partially penetrating well.

178 Figure 3b shows type curves for various degrees of penetration of the fracture into the aquifer
 179 (h_f/B ratio). In these examples, the fracture is located at the centre of the deepest aquifer
 180 ($z_f/B=0.5$), and there is no hydraulic conductivity anisotropy ($k_x=k_y=k_z$). As expected when h_f
 181 equals aquifer thickness (B), the solution is identical to the Gringarten et al. (1974) analytical
 182 solution until leakage from the upper aquifers starts. At the start of pumping, derivative
 183 curves follow the half-unit slope that shows linear flow from the aquifer to the fracture, before
 184 decreasing and following a negative slope tending to $-1/2$ that corresponds to ellipsoidal flow
 185 because of the partial entry of the fracture into the aquifer (low h_f/B ratios, curve 5 in Fig. 3b
 186 for instance). For intermediate times, the derivative curves form a first plateau corresponding
 187 to radial flow into the lower pumped aquifer ($s_D'=0.5$). Later, they have a 'V' shape
 188 characterizing leakage from the upper aquifers. Finally, for very late stages of pumping,
 189 derivative curves form a second plateau whose value depends on the transmissivity values of
 190 both upper and lower aquifers ($s_D'=T/2(T+T_0)$).

191 **3.2 Inclined fractures**

192

193 *3.2.1 Pumping at the centre of a single inclined fracture*

194 Figure 4a presents the conceptual model. The aquifer is characterized by transmissivity T and
 195 a storage coefficient S , and is anisotropic (k_x , k_y , k_z). It is overlain by a leaky aquifer of
 196 thickness B' and hydraulic conductivity k' . This conceptual aquifer model is similar to that
 197 proposed by Hantush (1961) and assumes that the aquitard does not react to pumping (infinite
 198 storage). The fracture crosscuts the aquifer, and is characterized by length L , width l , and
 199 angle θ with the vertical axis. The analytical solution of drawdown for pumping in a fracture
 200 in such an aquifer is found by integrating the point-source analytical solution given by Hunt
 201 (2005) on the fracture plane:

$$202 \quad s(x, y, z, t) = \frac{1}{2\pi T} \int_{-L/2}^{L/2} \int_{-l/2}^{l/2} q(\xi, \vartheta) \sum_{n=1}^{\infty} \frac{\cos(\alpha_n \xi) \cos(\alpha_n z/B)}{1 + \frac{\sin(2\alpha_n)}{2\alpha_n}} W\left(\frac{(r/B)^2}{4t_D}, \alpha_n \frac{r}{B} \sqrt{k_z/k_{xy}}\right) d\xi d\vartheta \quad \text{Eq.8}$$

203 where r is the radial distance to the well, B is aquifer thickness, k_z/k_{xy} its vertical anisotropy in
 204 hydraulic conductivity, t_D a dimensionless time, $q(\xi, \vartheta)$ the pumping rate per unit area, α_n the
 205 root of an equation, and $W(a, b)$ the Hantush leaky-aquifer well function; all parameters are
 206 explained in Appendix B.

207 As before, this equation is solved numerically by dividing the fracture in small elements along
 208 both L and l while assuming a uniformly distributed pumping rate per unit area (similarly to
 209 Eq.2; i.e. $q(\xi, \vartheta) = Q/(lL)$). Then the principle of superposition is applied for computing the
 210 drawdown at any location into the aquifer. Therefore, the solution yields:

$$211 \quad s(x, y, z, t) = \frac{Q}{2\pi T \cdot M \cdot p} \sum_{j=1}^p \sum_{i=1}^M \sum_{n=1}^{\infty} \frac{\cos(\alpha_n z_{i,j}/B) \cos(\alpha_n z/B)}{1 + \frac{\sin(2\alpha_n)}{2\alpha_n}} W\left(\frac{(r_{i,j}/B)^2}{4t_D}, \alpha_n \frac{r_{i,j}}{B} \sqrt{k_z/k_{xy}}\right) \quad \text{Eq.9}$$

212 where $r_{i,j}$ is the radial distance between the point (x, y, z) and the i, j^{th} element, and M and p
 213 are the number of segments along L and l .

214 Figure 4b presents type curves of Eq.9 for various values of θ . For this example, the aquifer
 215 has a vertical anisotropy of 10.0 ($k_{xy}/k_z=10$) and we ignore the leaky aquifer ($k'/B'=0$). The
 216 fracture is located at the centre of the aquifer ($z/B=0.5$) and its width equals the aquifer
 217 thickness ($l=B$).

218 As expected, the results show that Eq.9 is identical to the benchmark solutions: the Gringarten
 219 et al. (1974) solution for a vertical rectangular fracture when $\theta=0^\circ$, and the Thiéry (1980)

220 solution for a horizontal rectangular fracture located at the centre ($z_f/B=0.5$) of the aquifer,
221 when $\theta=90^\circ$. Derivative curves show that, regardless of the θ value, the early stages of
222 pumping always describe linear flow within the fracture (half-unit slope of derivative curves).
223 Later, derivatives create a hump, more or less pronounced according to the value of θ . This
224 behaviour corresponds to the transition from flow perpendicular to the fracture—controlled by
225 the average hydraulic conductivity normal to the fracture plane—to horizontal flow controlled
226 by the horizontal aquifer transmissivity. In some cases (e.g. $\theta=90^\circ$), the derivatives follow a
227 near-negative half-unit slope characterizing ellipsoidal flow because of the partial penetration
228 of the fracture into the aquifer. Note also that increasing the k_{xy}/k_z ratio with $\theta \rightarrow 90$, will result
229 in a more pronounced hump of the derivative curve, because of the increased resistance to
230 flow induced by the low vertical hydraulic conductivity. When the fracture is vertical and
231 fully penetrates the aquifer ($\theta=0^\circ$), the hump disappears as the drawdown no longer depends
232 upon the vertical anisotropy in hydraulic conductivity. For the late stages of pumping,
233 derivatives form a plateau characterizing the radial flow induced by flow towards the fracture
234 at the aquifer scale ($s_D=0.5$). As mentioned before, in experimental data the plateau value
235 will depend on the horizontal transmissivity of the aquifer.

236 3.2.2 Pumping intersecting two fractures – Pumping in the fractures only

237 One of the flexibilities inherent in our analytical/numerical solution is the possibility to
238 consider several fractures with different locations and orientations intersecting the pumping
239 well. Figures 5 and 6 show an example of application where two fractures intersect the well.
240 The characteristics of fracture 1 are L : 10 m; l : 20 m, θ : 20° , and z_f : 13 m, and of fracture 2 L :
241 10 m; l : 45 m, θ : 100° , and z_f : 5 m. As before, only fractures are pumped (uniform flux).

242 As shown previously, fractures with different orientations and locations are characterized by
243 different flow behaviours. Consequently, their resistance to flow is different; this implies,
244 when several fractures are pumped simultaneously, that the flowrates from each fracture vary
245 over time. Therefore, the flowrate contributions of each fracture to the total pumping rate of
246 the well have to be considered before evaluating the average drawdown in the well. To solve
247 this problem, we again use the principle of superposition, i.e. the drawdown in a particular
248 location is the sum of drawdown values of several pumping sources. This allows computing
249 the flowrates of each fracture from the combination of dimensionless pumping-source
250 solutions (see similar work in Lolon et al. 2008 and Lashgari et al., 2014). However, the
251 boundary conditions of each individual solution do not include perturbation due to the

252 presence of the other pumping source, which is the only approximation of this solution. Note
 253 that this approximation is the same as the one used for modelling drawdown (with analytical
 254 solutions) created by several pumping wells.

255 As the well is screened in front of each fracture only and as it intersects the fracture centre,
 256 the average dimensionless drawdown in the well, s_{Dtot} , can be derived from the following
 257 equations:

$$258 \quad s_{Dtot}Q = \frac{1}{2}[q_1(s_{D1} + s_{D12}) + q_2(s_{D2} + s_{D21})] \quad \text{Eq.10a}$$

$$259 \quad Q = q_1 + q_2 \quad \text{Eq.10b}$$

260 where q_1 and q_2 are flowrates from fractures 1 and 2, Q is the total pumping flowrate. s_{D1} and
 261 s_{D2} are the dimensionless drawdown values at the centre of fractures 1 and 2, s_{D12} is the
 262 dimensionless drawdown induced by fracture 1 at the centre of fracture 2, and s_{D21} , the one
 263 induced by fracture 2 at the centre of fracture 1.

264 At the intersection between fracture 1 and the well, the drawdown is:

$$265 \quad S_{F1} = q_1s_{D1} + q_2s_{D21} \quad \text{Eq.10c}$$

266 and at the intersection between fracture 2 and the well:

$$267 \quad S_{F2} = q_2s_{D2} + q_1s_{D12} \quad \text{Eq.10d}$$

268 Assuming equal drawdown at the centre of both fractures ($S_{F1}=S_{F2}$), thus implicitly assuming
 269 uniform drawdown at the well location, and solving the system of equations provided by Eqs.
 270 10, one will find solutions for s_{Dtot} , q_1 and q_2 :

$$271 \quad s_{Dtot} = \frac{1}{2}(A + BC)/(1 + C) \quad \text{Eq.11a}$$

$$272 \quad q_1 = Q/(1 + C), \text{ or } q_{D1} = 1/(1 + C) \quad \text{Eq.11b}$$

$$273 \quad q_2 = QC/(1 + C), \text{ or } q_{D2} = C/(1 + C) \quad \text{Eq.11c}$$

274 with $A = (s_{D1} + s_{D12})$, $B = (s_{D2} + s_{D21})$ and $C = (s_{D1} - s_{D12})/(s_{D2} - s_{D21})$, and dimensionless
 275 flowrate $q_{Di} = q_i/Q$.

276 For the computation, drawdown values s_{D1} , s_{D2} , s_{D12} and s_{D21} were calculated separately with
 277 Eq.9 while assuming a unit flow rate ($Q=1$).

278 Figures 6a and b present the resulting drawdown and the flowrate contributions of each
 279 fracture, and figures 6c, d, e and f show drawdown distribution in the vertical plane at
 280 different times. In the early stages of pumping, the drawdown derivative is characterized by
 281 the classical half-unit slope corresponding to flow from the aquifer to both fractures
 282 ($t < 0.8$ min; Fig. 6a and c). In the beginning, both fractures contribute almost equally to the
 283 total pumping rate (Fig. 6b), though in the very early stage ($t \leq 0.1$ min) fracture 2 contributes
 284 slightly more because of its larger area. However, very rapidly flowrate from fracture 1
 285 becomes dominant as its orientation offers less restriction to flow compared to fracture 2,
 286 which is oriented almost orthogonal to the hydraulic conductivity axis of the lower aquifer.
 287 Between 0.1 and 100 min, the derivative curve shows transitional flow behaviour, followed
 288 by a near $-1/2$ slope ($100 < t < 2000$ min) that characterizes the ellipsoidal flow towards both
 289 fractures as drawdown starts to progress in the whole aquifer (Fig. 6d). Later
 290 ($2000 < t < 7000$ min), the derivative stabilizes, forming a plateau characterizing radial flow
 291 from the aquifer to both fractures (Fig. 6e) whose value depends on the aquifer transmissivity.
 292 For the last stages ($t > 7000$ min), the derivative decreases because of leakage from the top
 293 aquitard (Fig. 6f). At the end of pumping, fracture 1 (the smallest; 200 m^2) contributes 74% of
 294 the total flow rate while fracture 2, the largest (450 m^2), contributes only 26% (Fig. 6b).

295 As a final point, this drawdown curve shows the difficulty of identifying the contribution of
 296 more than one fracture on a diagnosis plot, as this looks similar to one of the cases presented
 297 in Fig. 4 until leakage appears. In this case, only drawdown observation on other wells in the
 298 aquifer and/or few data on the geometry of fractures can provide unequivocal information
 299 about these fractures.

300 3.2.3. *Pumping intersecting two fractures – pumping in the fractures and in the* 301 *aquifer through the well itself*

302 This last case is similar to the sketch presented on Figure 5, but pumping affects both
 303 fractures as well as the aquifer directly through a screened portion of the well. The well
 304 intersects the centres of both fractures. As for the previous case, flowrate contributions of
 305 each individual pumping source (the two fractures and the well) have to be evaluated before
 306 computing the average dimensionless drawdown, s_{Dtot} , in the well:

$$307 \quad s_{D_{tot}} Q = q_1 s_{D1W} + q_2 s_{D2W} + q_w s_w \quad \text{Eq.12a}$$

$$308 \quad \text{with } Q = q_1 + q_2 + q_w \quad \text{Eq.12b}$$

309 where q_1 and q_2 are flowrates from fractures 1 and 2, q_w the flowrate from the aquifer through
 310 the well and Q the total pumping flowrate. s_{D1W} and s_{D2W} are the dimensionless drawdown
 311 values induced by fractures 1 and 2 in the screened well interval. s_{D1W} and s_{D2W} are the
 312 average drawdown values along the screened interval, computed by integrating the analytical
 313 solution (Eq.8) between the upper and lower limits of the pumped section. s_w is the
 314 dimensionless drawdown induced by pumping in the screened interval (Hunt's (2005)
 315 analytical solution).

316 At the intersection between fractures and the well, the drawdown in the well has to be
 317 identical to that from the fractures. Therefore, at the intersection between the well and fracture
 318 1:

$$319 \quad q_w s_w = q_1 s_{D1} + q_2 s_{D21} \quad \text{Eq.12c}$$

320 and between the well and fracture 2:

$$321 \quad q_w s_w = q_2 s_{D2} + q_1 s_{D12} \quad \text{Eq.12d}$$

322 with s_{D1} and s_{D2} the dimensionless drawdown values at the centre of fractures 1 and 2, s_{D12} the
 323 dimensionless drawdown induced by fracture 1 at the centre of fracture 2, and s_{D21} the one
 324 induced by fracture 2 at the centre of fracture 1.

325 Assuming equal drawdown at the centre of both fractures (Eq.12c=Eq.12d) in the well in front
 326 of both fractures, and solving the system of equations provided by Eqs.12, solutions are found
 327 for $s_{D_{tot}}$, q_1 , q_2 and q_w :

$$328 \quad s_{D_{tot}} = (s_{1W} + C s_{2W} + D) / (1 + C + D/s_w) \quad \text{Eq.13a}$$

$$329 \quad q_1 = Q / (1 + C + D/s_w), \text{ or } q_{D1} = 1 / (1 + C + D/s_w) \quad \text{Eq.13b}$$

$$330 \quad q_2 = QC / (1 + C + D/s_w), \text{ or } q_{D2} = C / (1 + C + D/s_w) \quad \text{Eq.13c}$$

$$331 \quad q_w = QD / [s_w (1 + C + D/s_w)], \text{ or } q_{DW} = D / [s_w (1 + C + D/s_w)] \quad \text{Eq.13d}$$

332 where C is defined before (Eqs.11), $D = (s_{D1}s_{D2} - s_{D12}s_{D21}) / (s_{D2} - s_{D21})$, and $q_{Di} = q_i / Q$ are
 333 the dimensionless flowrates.

334 Drawdown computations and flowrate of each pumping source were made for a well fully
 335 penetrating an aquifer with characteristics and fractures as sketched on Figure 5. Drawdown
 336 values were computed separately with Eq.9 for the fractures and with the Hunt (2005)
 337 analytical solution for the well (vertical, fully penetrating the aquifer and uniform flux
 338 distribution). Figures 7a and b show the results and Figures 7c, d, e and f the drawdown
 339 distribution (on a vertical axis) at different times.

340 The drawdown behaviour is very different to the case where only the fractures are pumped
 341 (Fig. 6a). At the start of pumping (see also Fig. 7c), the half-unit slope persists as the
 342 contribution of the aquifer through the well in terms of flowrate is low compared to that from
 343 the fractures (Fig. 7b). After 1 to ~1000 minutes, the derivative curve slightly decreases,
 344 following a gentle near-negative slope. This period describes the transitional flow regime
 345 during which the contribution through the well starts to contribute significantly to the total
 346 flowrate. During this period, ellipsoidal flows from fractures still exist (Fig. 7d), but are
 347 masked by the well contribution. Between 1000 and ~7000 minutes (Fig. 7e), the derivative
 348 curve forms a plateau corresponding to near-radial flow induced by the three pumping
 349 sources. At the end of pumping, the well contributes up to 52% of the total flow rate, fracture
 350 1 up to 36%, while fracture 2 yields only 12%.

351 The high flowrate contribution from the aquifer to the well, here equivalent to both fractures,
 352 is due to two reasons: *i*) The medium to high transmissivity of the aquifer ($T=10^{-4} \text{ m}^2/\text{s}$), or
 353 more exactly its diffusivity; and *ii*) Because the screened interval covers the entire aquifer
 354 thickness. If the aquifer transmissivity had been very low (e.g. $<10^{-8} \text{ m}^2/\text{s}$), or if the well had
 355 been screened near the fractures, the aquifer contribution through the well would have been
 356 negligible or drastically less, creating an average drawdown in the well similar to the one
 357 observed on Fig. 6. Appendix C provides an intermediate figure where the well is screened in
 358 front of both fractures only. Fractures and aquifer properties are those of Fig. 5 with a well-
 359 screened interval of 7 m. Drawdown and derivative behaviour are different from the previous
 360 case, and closer to those of Fig. 6 because of the lower contribution of the aquifer through the
 361 well to the total pumping flowrate (at the end of pumping, well: 20%, fracture1: 56% and
 362 fracture 2: 22%).

363 This result shows that where an aquifer has significant transmissivity, the fracture signature
364 (i.e. the half-unit derivative slope in the early stages of pumping) may be very short, and even
365 not seen. This may explain why pumping tests in fractured media commonly are not
366 interpreted with solutions invoking discrete fractures intersected by the well, but with
367 homogeneous aquifer solutions (e.g. dual-porosity models), even if fractures were seen on
368 field data (e.g. Taylor and Howard, 2000; Maréchal et al., 2004; Dewandel et al., 2011, 2017).

369

370 **4. Conclusions**

371 Integrating the well-known Theis analytical solution (1935) along a fracture axis is
372 mathematically identical to the equally well-known solution of Gringarten et al. (1974) for a
373 uniform-flux fracture fully penetrating an aquifer, obtained with Green's function and the
374 product solution method. Though this result was mathematically expected (Green's function
375 represents the distribution of instantaneous point-sources over length, area or volume), this
376 had never before been demonstrated or highlighted as far as we are aware. This implies that
377 any line- or point-source solution integrated along the fracture plane can be used for
378 computing flow through a fracture. In view of the large number of existing analytical
379 solutions, this provides a very wide range of applications for this generic analytical solution,
380 and thus helps modelling discrete fractures that are intersected and pumped by a well, in most
381 hydrogeological settings.

382 We give several theoretical examples for dual-porosity or multilayer aquifer types, even when
383 the pumping well intersects several fractures, and for pumping both in fractures and/or
384 directly in the aquifer through the screened interval of a well. Where several sources are
385 pumped (the aquifer through the well and/or several fractures), solutions of flowrate
386 contributions of each individual pumping source are given and can be extended to any number
387 of fractures.

388 As expected when the conceptual hydrogeological models respect the theoretical assumptions
389 of benchmark analytical solutions (here for pumping in a vertical or a horizontal fracture), the
390 proposed solutions do not show significant differences between models.

391 Where a fracture is vertical and fully intersects the aquifer, the classical flow regime of a
392 fracture can be recognized regardless of the conceptual aquifer model used. However, in the
393 case of an inclined fracture, we show that flow behaviour is not unique. In that case, it

394 depends on the screened-interval length of the well, as well as on whether only the fracture, or
395 both the fracture and the surrounding aquifer are screened. Even, when aquifer transmissivity
396 (or diffusivity) is not low, the fracture signature (the classical half-unit slope on a derivative
397 drawdown curve) can be masked by the contribution of the screened interval of the well.

398 Our work is based on the assumption that flux is uniformly distributed along the fracture and
399 the screened interval of the well. Even if this hypothesis is the closest approximation for an
400 infinite conductivity fracture –it is nonetheless identical at early stages of pumping- solutions
401 for infinite conductivity fracture can be derived from uniform flux analytical solutions
402 (Gringarten et al., 1974). This can be computed by dividing the fracture into small sections,
403 each with uniform flux per unit area, and then by evaluating the flow contribution of each
404 section to arrive at identical drawdown values along the entire fracture, including the screened
405 interval of the well if it is pumped as well.

406 In addition to the possibility of using any line- or point-source solutions, our generic
407 analytical solution offers several other advantages. For example, by using the superposition
408 principle (image well theory) it allows developing solutions where no-flow and constant-head
409 boundaries are not necessarily parallel to the fracture directions. Currently available analytical
410 solutions, however, assume fractures parallel or at right angle to boundaries. In the case of a
411 well field, our solution allows computing the overall drawdown created by several pumping
412 wells intersecting fractures with various geometries, or when only some of the wells intersect
413 such fractures, the others directly pumping the aquifer. Finally, though we established
414 analytical solutions for rectangular fractures, these solutions can also be developed for any
415 other geometrical configuration.

416 From a practical point view, the proposed analytical solutions and few diagnosis plots as
417 presented provide additional information on flow behaviour and drawdown in fractured
418 media. Their application to real field data is expected soon to characterize fracture geometries,
419 particularly in the case of fractured thermo-mineral aquifers (Maréchal et al., 2014). This
420 should help defining, among other points, the location of a well within a given deep fracture,
421 and then to establish if this fracture extends very deep or not. In addition, it should help
422 characterizing fractures of the weathered Stratiform Fractured Layer of hard rock aquifers
423 (Lachassagne et al., 2011) as well as their relationships with overlying saprolite (leakage
424 effects).

425

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430

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- 525

526 **Figure captions**

527 **Figure 1.** Conceptual sketch of a well intercepting a single vertical fracture; it corresponds to
 528 the analytical solution of Gringarten et al. (1974) (plan view).

529 **Figure 2.** A single fully penetrating fracture in a dual-porosity aquifer (dual-porosity
 530 conceptual model: Moench, 1984), pumping at the centre of the fracture. Type curves for
 531 various λ coefficients ($\lambda = K_m r^2 / (K_f b_m)$). Dimensionless drawdown $-s_D$: plain curves and
 532 derivatives $-s_D'$: dotted curves. Circles correspond to the Gringarten et al. (1974) solution.
 533 Dual-porosity model parameters: K_f and K_m , fracture and matrix hydraulic conductivity, b_m :
 534 block size, r : radial distance from the well, s_f and s_m : fracture and matrix storage, sk_f : fracture
 535 skin.

536 **Figure 3.** A single partially entering vertical fracture in a multilayer aquifer. Fracture in the
 537 deepest layer, and pumping at the centre of the fracture. a) Conceptual model with aquifer
 538 parameters (see text for explanation). b) Type curves for various h_f/B ratios. Circles
 539 correspond to the Gringarten et al. (1974) analytical solution.

540 **Figure 4.** Pumping at the centre of an inclined fracture in a leaky aquifer system. a)
 541 Conceptual model with aquifer parameters (see text for explanation). b) Type curves for
 542 various angles of the fracture with the vertical, θ , and $k'/B'=0$ (no-leakage). Dots correspond
 543 to the Gringarten et al. (1974) and Thiéry (1980) analytical solutions.

544 **Figure 5.** Two inclined fractures in a leaky aquifer system: model parameters.

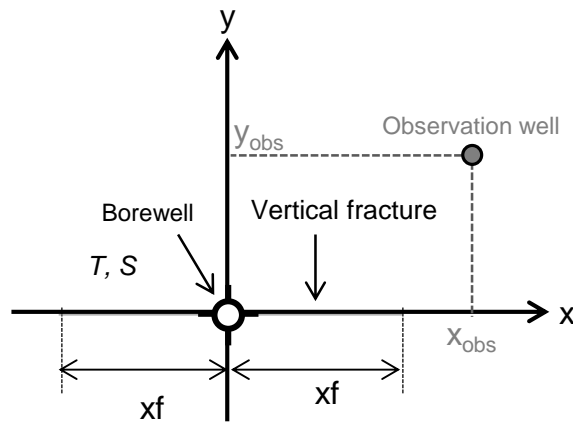
545 **Figure 6.** Two inclined fractures in a leaky aquifer system; the well is screened in front of
 546 both fractures only. a) Average drawdown and its derivative at the well. b) Dimensionless
 547 flowrate contributions of each fracture. c), d), e) and f) Drawdown distributions at various
 548 times along the x-axis. Pumping rate is $1 \text{ m}^3/\text{h}$.

549 **Figure 7.** Two inclined fractures in a leaky aquifer system, pumping in the fractures and in
 550 the aquifer through a fully penetrating well. a) Average drawdown and its derivative at the
 551 well. b) Dimensionless flowrate contributions of each fracture and of the aquifer through the
 552 well. c), d), e) and f) Drawdown distributions at various times along the x-axis. Pumping rate
 553 is $1 \text{ m}^3/\text{h}$.

554 Caption for the figure in Appendix C: drawdown and its derivative (a) and flowrates (b)
 555 behaviour for a well screened in front of both fractures (length: 7 m); both fractures and the

556 aquifer though the screened-interval are pumped. Fractures and aquifer properties are the ones
 557 of Figure 5. Pumping rate is $1 \text{ m}^3/\text{h}$.

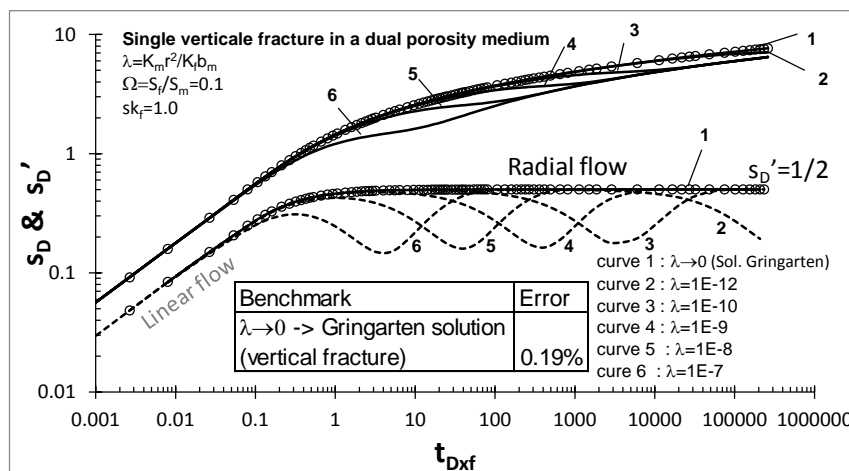
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559

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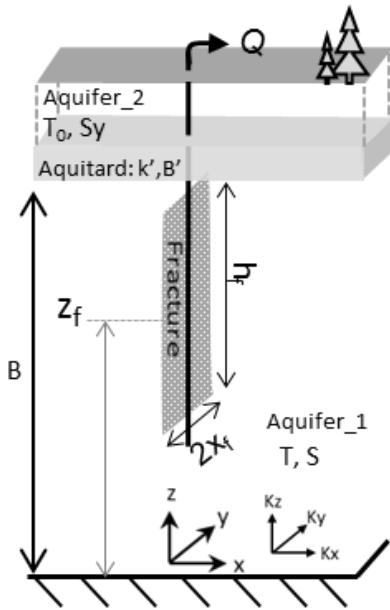
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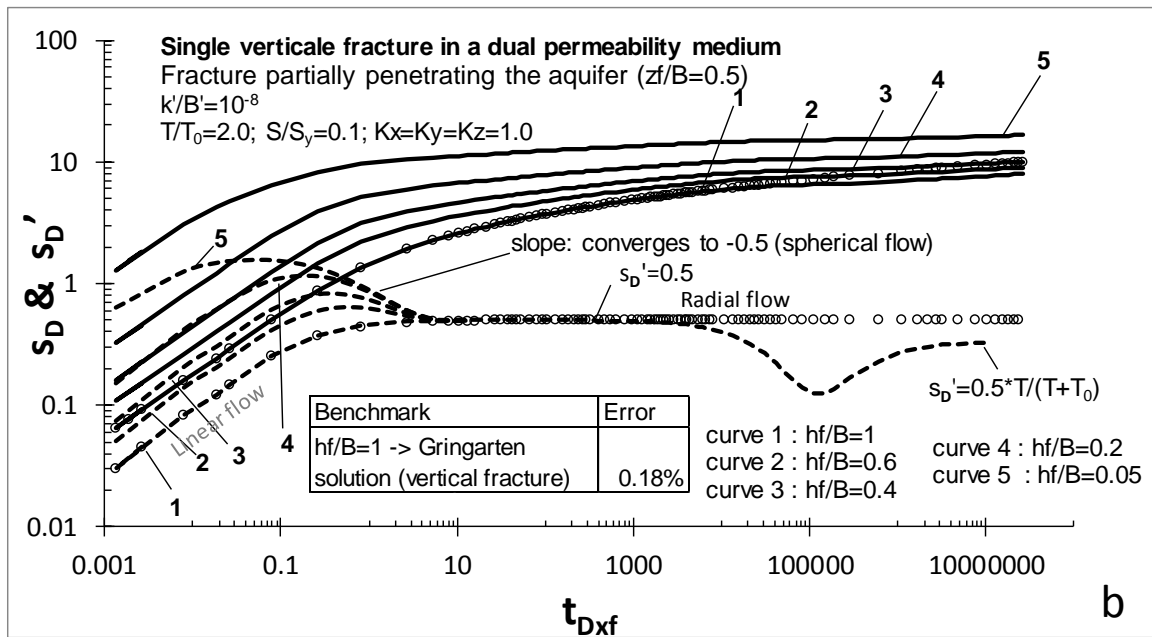
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 566 various λ coefficients ($\lambda = K_m r^2 / (K_f b_m)$). Drawdown $-s_D$: plain curves and derivatives $-s_D'$:
 567 dotted curves. Circles correspond to the Gringarten et al. (1974) solution. Dual-porosity
 568 model parameters: K_f and K_m , fracture and matrix hydraulic conductivity, b_m : block size, r :
 569 radial distance from the well, s_f and s_m : fracture and matrix storage, sk_f : fracture skin.

570



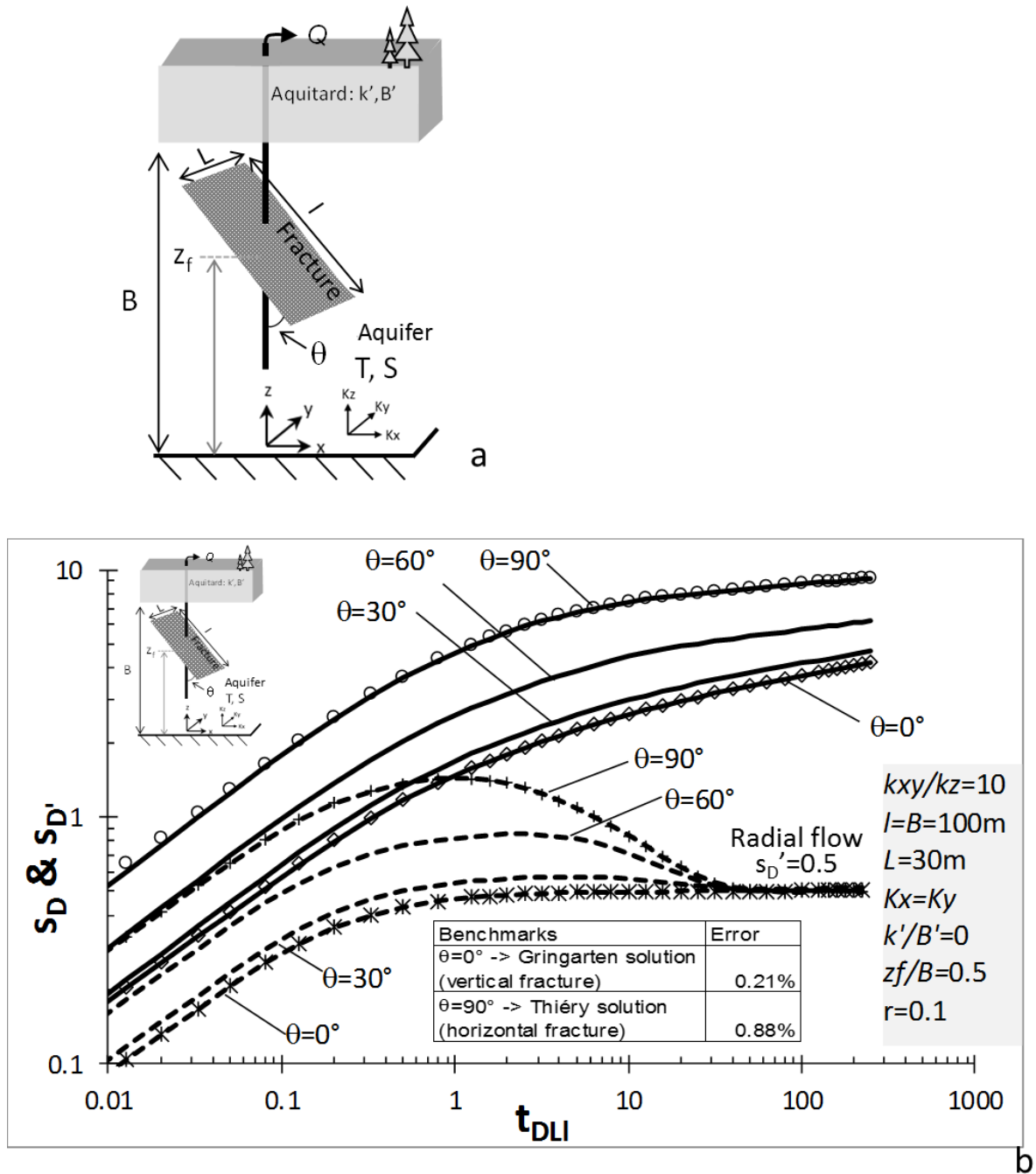
a



b

571

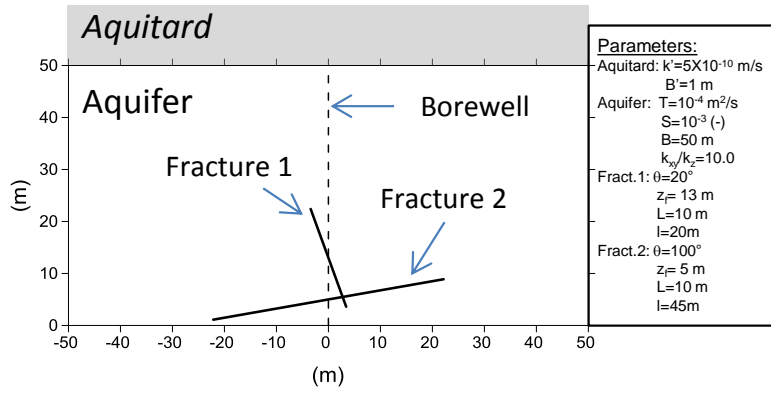
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576

577 **Figure 4.** Pumping at the centre of an inclined fracture in a leaky aquifer system. a)
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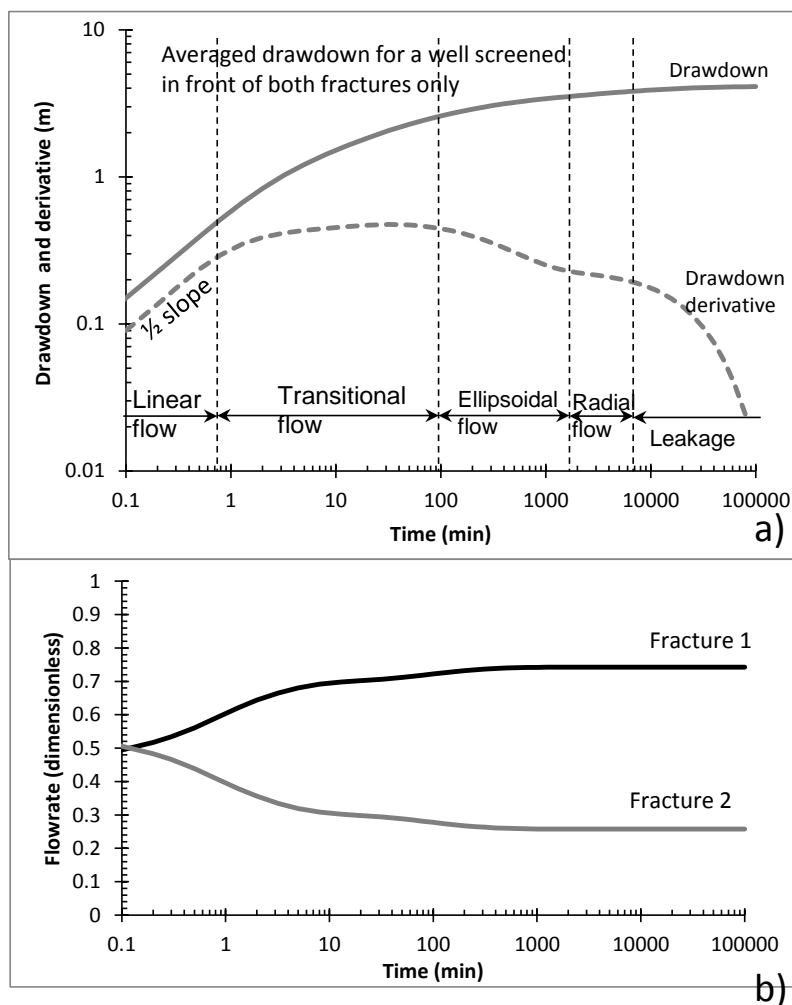
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583 **Figure 5.** Two inclined fractures in a leaky aquifer system: model parameters.

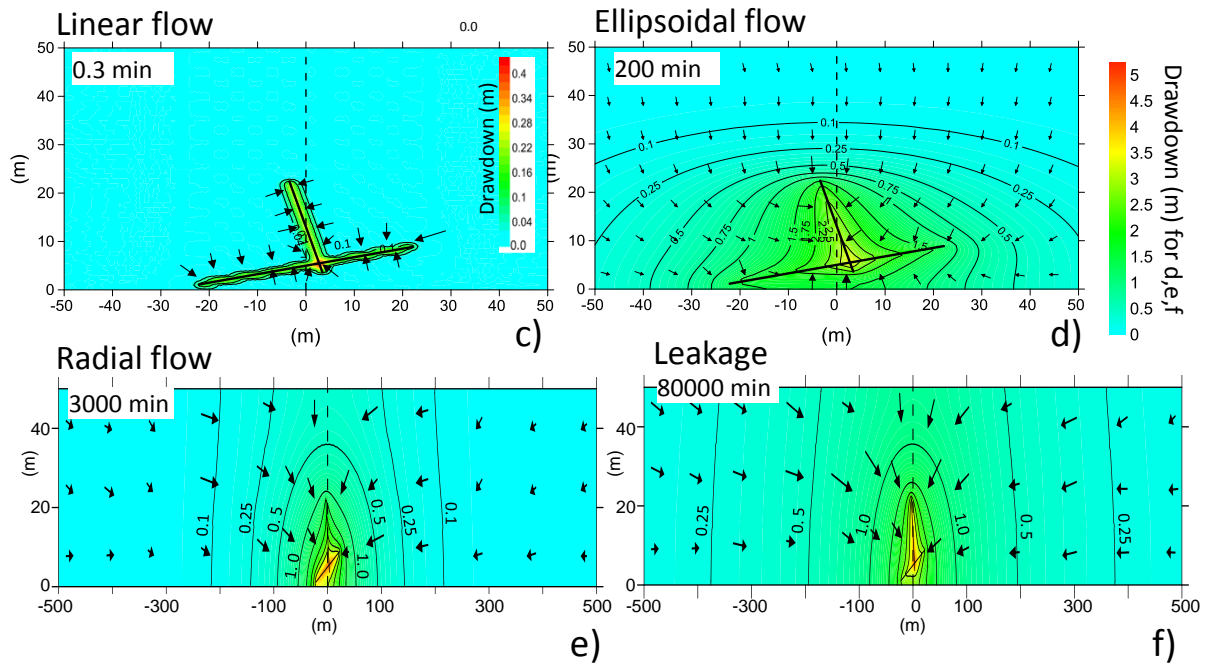
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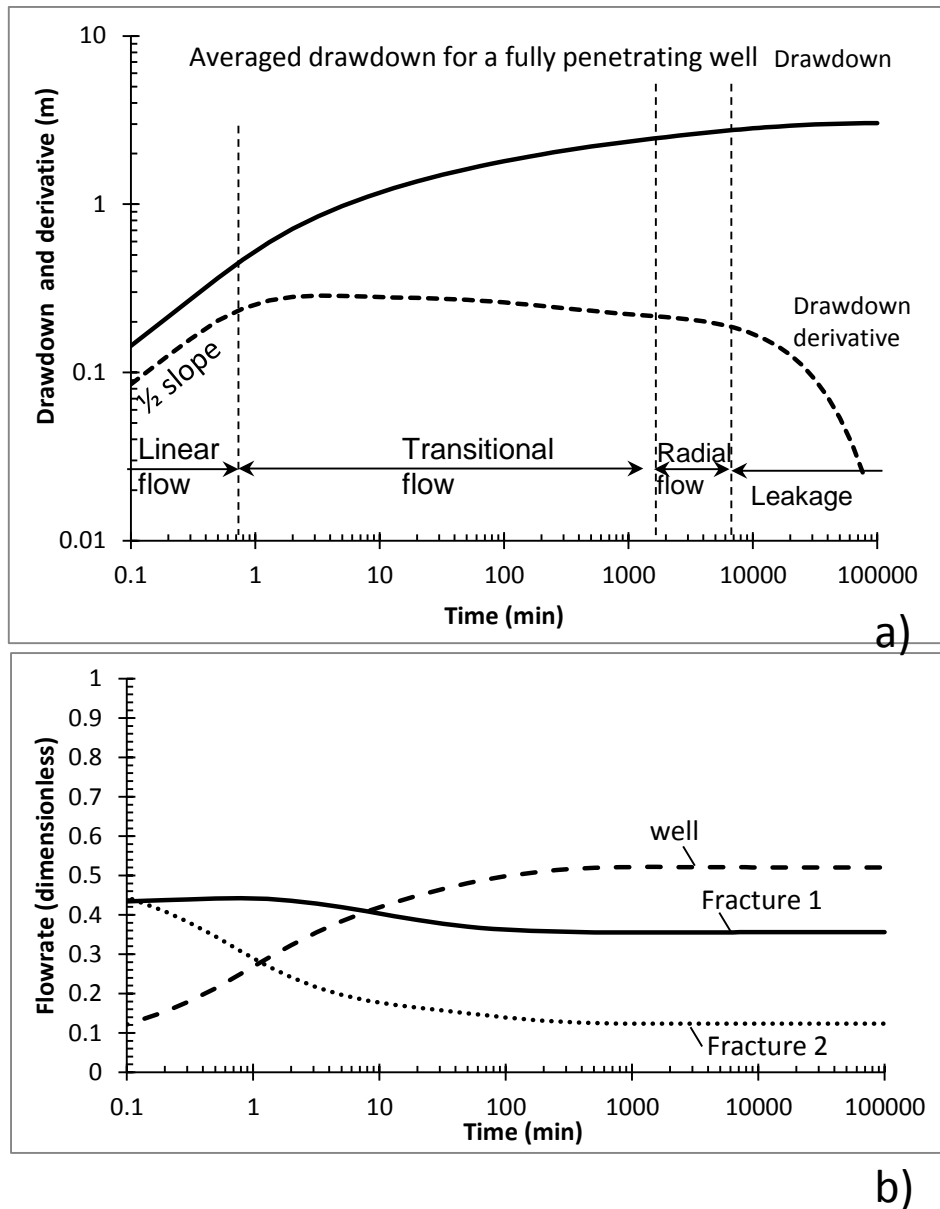
586 **Figure 6.** Two inclined fractures in a leaky aquifer system; the well is screened in front of
 587 both fractures only. a) Average drawdown and its derivative at the well. b) Dimensionless
 588 flowrate contributions of each fracture. Pumping rate is 1 m³/h.

589



590

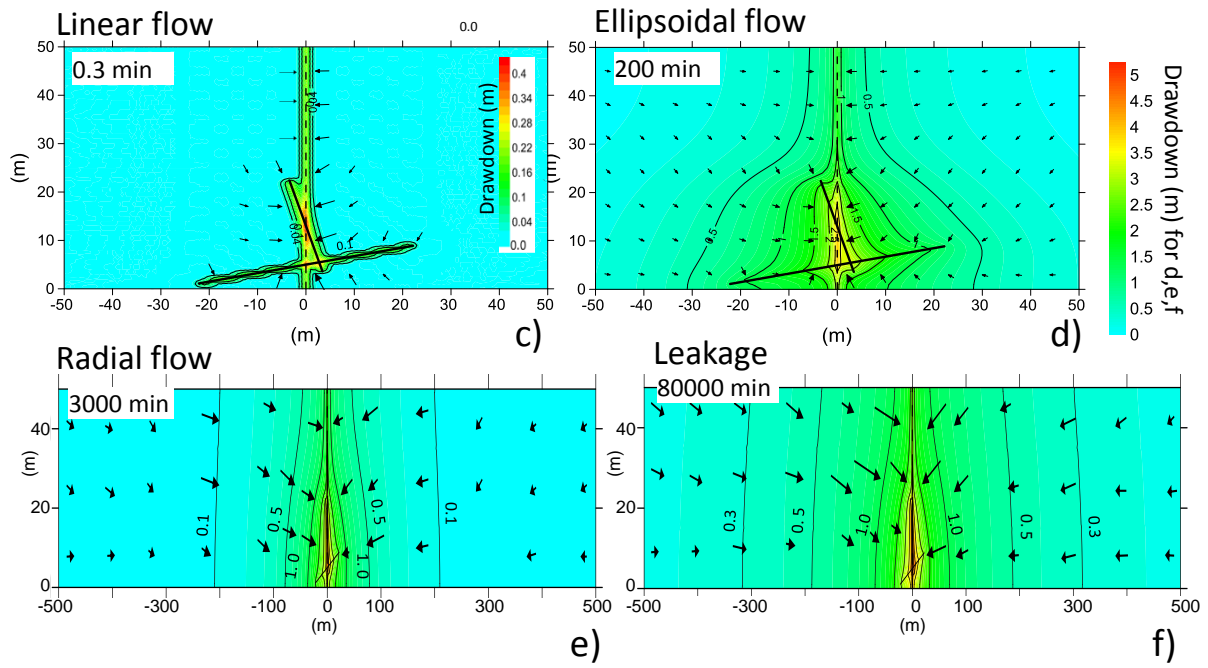
591 **Figure 6. cont'd.** c), d), e) and f) Drawdown distributions at various times along the x-axis.



592

593 **Figure 7.** Two inclined fractures in a leaky aquifer system, pumping in the fractures and in
 594 the aquifer through a fully penetrating well. a) Average drawdown and its derivative at the
 595 well. b) Dimensionless flowrate contributions of each fracture and, of the aquifer through the
 596 well. Pumping rate is $1 \text{ m}^3/\text{h}$.

597



598

599 **Figure 7. cont'd.** c), d), e) and f) Drawdown distributions at various times along the x-axis.

600

601 **Appendices:**602 **Appendix A: Dimensionless drawdown and time**603 Dimensionless drawdown, s_D :

604
$$s_D = \frac{2\pi T}{Q} s$$

605 With s , the drawdown, T , the transmissivity, and Q , the pumping rate.606 Dimensionless time, t_{Dxf} and t_{DLI} :607 - For a vertical fracture fully penetrating the aquifer, t_{Dxf} :

608
$$t_{Dxf} = \frac{tT}{Sx_f^2};$$
 with x_f , the half-fracture length, t , the time and S , the storage coefficient of

609 the aquifer.

610 - For a fracture partially penetrating the aquifer, t_{DLI} :

611
$$t_{DLI} = \frac{tT}{S(L/2.l/2)};$$
 with L and l , the length and the width of the fracture.

612

613 **Appendix B: point-source solution of Hunt (2005)**614 Point-source (or sink source) solution for the aquifer described in Figure 4 (Hunt, 2005), from
615 which 3-D anisotropy in hydraulic conductivity has been implemented:

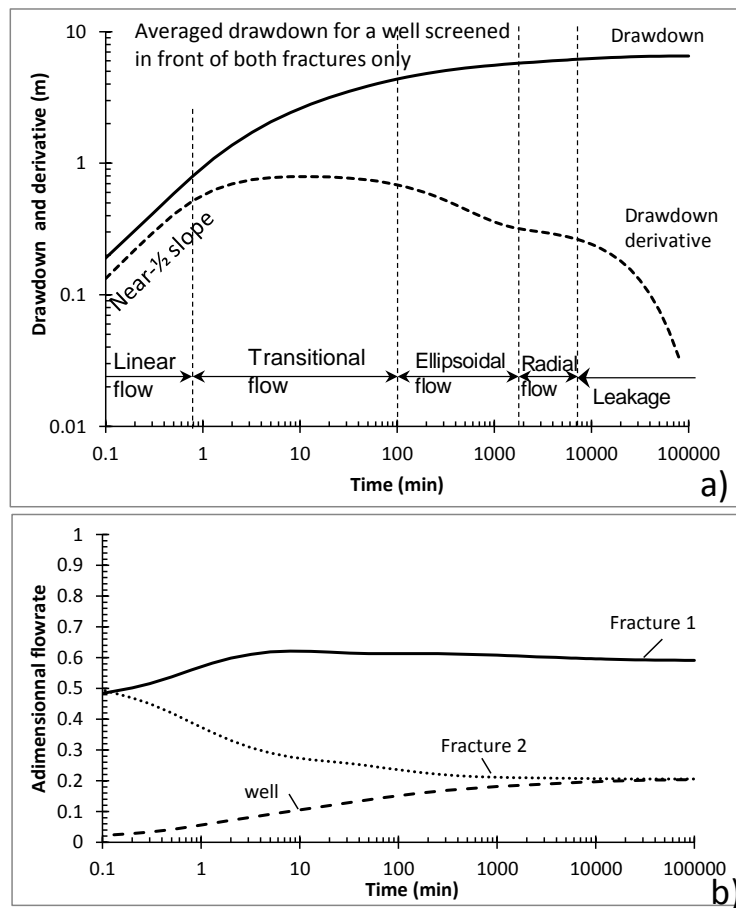
616
$$s_{\text{sink}}(x_D, y_D, z_D, t_D) = \frac{Q}{2\pi T} \sum_{n=1}^{\infty} \frac{\cos(\alpha_n Z/B) \cos(\alpha_n z_D)}{1 + \frac{\sin(2\alpha_n)}{2\alpha_n}} W\left(\frac{(r/B)^2}{4t_D}, \alpha_n \frac{r}{B} \sqrt{k_z/k_{xy}}\right)$$

617 where $x_D = \frac{x}{B} \sqrt{k_y/k_{xy}}$, $y_D = \frac{y}{B} \sqrt{k_x/k_{xy}}$, $z_D = \frac{z}{B}$, are dimensionless variables of the point618 location (x, y, z) ; k_x , k_y , k_z are the hydraulic conductivity along the x , y , z directions; Z is the z
619 coordinate of the point source where pumping Q takes place; k_{xy} is the horizontal hydraulic620 conductivity of the aquifer ($k_{xy} = \sqrt{k_x k_y} = T/B$); B is the aquifer thickness and $t_D = tk_{xy}/SB$;

621 t is time; α_n is the root of equation $\alpha_n \tan(\alpha_n) = \frac{k'/B'}{k_z/B}$, with k' and B' the hydraulic
 622 conductivity and the thickness of the leaky aquifer; $W(a, b)$ is the Hantush leaky-aquifer well
 623 function.

624

625 **Appendix C:** drawdown and its derivative (a) and flowrates (b) behaviour for a well screened
 626 in front of both fractures (length: 7 m); both fractures and the aquifer through the screened-
 627 interval are pumped. Fractures and aquifer properties are the ones of Figure 5. Pumping rate is
 628 $1 \text{ m}^3/\text{h}$.



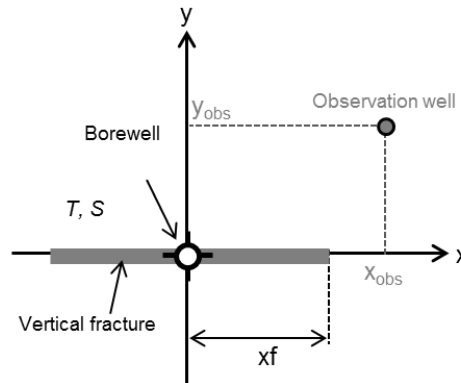
629

630

631 **Supplemental material:**

632 **Demonstrating that the well Theis function integrated along a fracture plane is identical**
 633 **to the analytical solution of pumping in a vertical fracture proposed by Gringarten et al.**
 634 **(1974) with uniform flux distribution.**

635 Definition conceptual model: the borewell intercepts a vertical fracture of length $2xf$ in a
 636 homogeneous and infinite aquifer of transmissivity, T , and storativity, S . The width of the
 637 fracture is assumed to be negligible.



638

639 **Notations**

640 $r^2 = (x - x_{obs})^2 + y_{obs}^2$ with x a point in the fracture ; $t_D = \frac{Tt}{x_f^2 S}$; $x_D = \frac{x_{obs}}{x_f}$; $y_D = \frac{y_{obs}}{x_f}$;

641 $u = \frac{r^2 \cdot S}{4Tt} \Rightarrow u = \frac{(x - x_{obs})^2 + y_{obs}^2}{4Tt} S = \frac{(x/x_f - x_D)^2 + y_D^2}{4t_D}$ and $a = u \cdot t_D = \frac{(x/x_f - x_D)^2 + y_D^2}{4}$

642

643 **Properties of the well function $W(u)$:**

644 $W(u) = E_1(u) = -E_i(-u) = \int_u^\infty \frac{e^{-y}}{y} dy$

645 With the following change of variable: $y = \frac{a}{\tau}$, it becomes that $\int_{a/t_D}^\infty \frac{e^{-y}}{y} dy = \int_0^{t_D} \frac{e^{-a/\tau}}{\tau} d\tau$

646 **Properties of the Erf function:**

647 $Erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du = -\frac{2}{\sqrt{\pi}} \int_{-x}^0 e^{-u^2} du$

648 $Erf(x) = -Erf(-x)$

649

650 **Well Theis function for a well**

651 The aquifer is pumped in a single vertical well fully penetrating the aquifer with a pumping
 652 rate q . The drawdown at a radial distance r from the well can be expressed as:

$$653 \begin{cases} s(r,t) = \frac{q}{4\pi T} W(u) = \frac{q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y} dy \\ u = \frac{r^2 S}{4Tt} \end{cases} \quad [Eq.1]$$

654

655 **Gringarten analytical solution (Gringarten et al., 1974)**

656 For a pumping in a single vertical fracture intersecting the pumped well, with uniform flux
 657 distribution, the drawdown can be expressed as:

$$658 s(x, y, t) = \frac{Q}{2\pi T} s_D \text{ with } s_D = \int_0^{t_D} e^{-\frac{y_D^2}{4t_D}} \left[\text{Erf}\left(\frac{1-x_D}{2\sqrt{\tau}}\right) - \text{Erf}\left(\frac{1+x_D}{2\sqrt{\tau}}\right) \right] \frac{\sqrt{\pi}}{4\sqrt{\tau}} d\tau \quad [Eq.2]$$

659 Where Q is the pumping rate. As Q is assumed to be uniformly distributed along the fracture,
 660 therefore:

$$661 Q = \int_{-x_f}^{+x_f} q(x) dx \Rightarrow q(x) = q = \frac{Q}{2x_f}$$

662 **Mathematical demonstration**

663 Integration of the Theis' solution along the fracture yields:

$$664 s_I(x, y, t) = \int_{-x_f}^{+x_f} s(x, y, t) dx = \frac{1}{4\pi T} \int_{-x_f}^{+x_f} q E_1\left(\frac{(x-x_{obs})^2 + y_{obs}^2}{4Tt}\right) S dx$$

665 with the simplified notation and using the property of the E_1 function, it becomes:

$$666 s_I = \frac{Q}{2\pi T} \frac{1}{4x_f} \int_{-x_f}^{+x_f} E_1\left(\frac{a}{t_D}\right) dx = \frac{Q}{2\pi T} \frac{1}{4x_f} \int_{-x_f}^{+x_f} \left(\int_{a/t_D}^{+\infty} \frac{e^{-u}}{u} du \right) dx = \frac{Q}{2\pi T} \frac{1}{4x_f} \int_{-x_f}^{+x_f} \left(\int_0^{t_D} \frac{e^{-a/\tau}}{\tau} d\tau \right) dx$$

$$667 \text{ which can be rewritten as : } s_I = \frac{Q}{2\pi T} \frac{1}{4x_f} \int_{-x_f}^{+x_f} \left(\int_0^{t_D} \frac{e^{-\left[\left(\frac{x}{x_f} - x_D\right)^2 + y_D^2\right]/4\tau}}{\tau} d\tau \right) dx$$

668 According to the Fubini theorem (i.e. τ and x are independent), the order of integration can be
 669 inverted, yielding to:

$$670 \quad s_I = \frac{Q}{2\pi T} \frac{1}{x_f} \int_{-x_f}^{+x_f} \left(\int_0^{t_D} e^{-y_D^2/4\tau} \frac{e^{-\left(\frac{x}{x_f} - x_D\right)^2/4\tau}}{4\tau} d\tau \right) dx = \frac{Q}{2\pi T} \frac{1}{x_f} \int_0^{t_D} \left(\int_{-x_f}^{+x_f} e^{-y_D^2/4\tau} \frac{e^{-\left(\frac{x}{x_f} - x_D\right)^2/4\tau}}{4\tau} dx \right) d\tau$$

671 and as $e^{-y_D^2/4\tau}$ does not depend on x , it can be rearranged as:

$$672 \quad s_I = \frac{Q}{2\pi T} \frac{1}{x_f} \int_0^{t_D} e^{-y_D^2/4\tau} \underbrace{\left(\int_{-x_f}^{+x_f} \frac{e^{-\left(\frac{x}{x_f} - x_D\right)^2/4\tau}}{4\tau} dx \right)}_{(a)} d\tau \tag{a} \tag{Eq.3}$$

673
 674 With the following change of variable: $v = \left(\frac{x}{x_f} - x_D\right) / 2\sqrt{\tau}$, the (a) term of [Eq.3] can be
 675 rewritten as :

$$676 \quad \int_{-x_f}^{+x_f} \frac{e^{-\left(\frac{x}{x_f} - x_D\right)^2/4\tau}}{4\tau} dx = \frac{x_f}{4\sqrt{\tau}} \int_{-(1+x_D)/2\sqrt{\tau}}^{(1-x_D)/2\sqrt{\tau}} 2e^{-v^2} dv \tag{Eq.4}$$

677 The right part of [Eq.4] can be decomposed into two terms related to the *Erf* function:

$$678 \quad \frac{x_f}{4\sqrt{\tau}} \int_{-(1+x_D)/2\sqrt{\tau}}^{(1-x_D)/2\sqrt{\tau}} 2e^{-v^2} dv = \frac{x_f}{4\sqrt{\tau}} \left(\int_{-(1+x_D)/2\sqrt{\tau}}^0 2e^{-v^2} dv + \int_0^{(1-x_D)/2\sqrt{\tau}} 2e^{-v^2} dv \right) = \frac{x_f \cdot \sqrt{\pi}}{4\sqrt{\tau}} \left[-Erf\left(\frac{(1+x_D)}{2\sqrt{\tau}}\right) + Erf\left(\frac{(1-x_D)}{2\sqrt{\tau}}\right) \right] \tag{Eq.5}$$

680 Combining [Eq.3] and [Eq.5], we obtain:

$$681 \quad s_I = \frac{Q}{2\pi T} \frac{1}{x_f} \int_0^{t_D} e^{-\frac{y_D^2}{4\tau}} \frac{x_f \sqrt{\pi}}{4\sqrt{\tau}} \left[-Erf\left(\frac{(1+x_D)}{2\sqrt{\tau}}\right) + Erf\left(\frac{(1-x_D)}{2\sqrt{\tau}}\right) \right] d\tau$$

$$682 \quad \text{And thus : } \boxed{s_I = \frac{Q}{2\pi T} \int_0^{t_D} e^{-\frac{y_D^2}{4\tau}} \left[Erf\left(\frac{(1-x_D)}{2\sqrt{\tau}}\right) - Erf\left(\frac{(1+x_D)}{2\sqrt{\tau}}\right) \right] \frac{\sqrt{\pi}}{4\sqrt{\tau}} d\tau} \tag{Eq.6}$$

683 [Eq.6] proves that the Theis analytical solution integrated along the fracture plane is exactly
 684 the same as the Gringarten analytical solution, for a vertical fracture with a uniform flux
 685 distribution (Eq. 2 or the Eq. 20 in Gringarten et al. 1974).

686

