Practical Inversion of Electric Resistivity in 3D from Frequency-domain Land CSEM Data

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Introduction

EM prospection are method of choice for many applications such as deep water or geothermal prospection because of their sensitivity to electrical resistivity and their potential to investigate at depths of 500m or even more. However, the investigated areas in europoe are usually urbanised and industrialised so high level of cultural noise prevents from the use of MT. Land CSEM is an alternative. But cost and logistical constrains may limits to the use of a small number of transmiter positions, often only one. The inversion of CSEM data in the near field using a single transmiter position suffers from critical sensitivity singularities due to the unsymmetric illumination. To overcome this problem we proposed an inversion framework adapted to this ill-conditioned inversion problem. The framework relies specificaly on a robust Gauss-Newton solver, on model parameter transformations and data reformulation under the form of pseudo-MT tensors. We describe here the modeling and inversion approach implemented in our code POLYEM3D and describe the framework proposed for its practical application. We illustrate its application on synthetic cases and then show the application of the process to a real CSEM dataset acquired for thermal water prospection at a few kilometer from a nuclear power plant in France.

An workflow for robust 3D CSEM inversion

The POLYEM3D code (Bretaudeau et al., 2016) used in this study relies on an hybrid semi-analytical/finite-volume modeling on irregular cartesian grid Streich (2009). The FV formulation provides a linear system:

$$\mathbf{A}(\rho,\omega)\,\mathbf{E} = \mathbf{b}. \tag{1}$$

where $\mathbf{A}$ is the finite-volume operator matrix, ρ a 3D resistivity distribution, $\mathbf{E}$ the 3D electric field and $\mathbf{b}$ the source term.
The computed data $d^c_{s,r}$ (component c of the electric and/or magnetic field at each receiver r generated by the source s) can be expressed as:

$$d^c_{s,r} = \wp^c_r\,\mathbf{E_s} \tag{2}$$

where $\wp^c_r$ is a restriction operator that extract the value of the component of the field from the 3D electric field computed on the whole grid. It contains interpolation operators and curl operator for magnetic field.

Inversion of EM fields is achieved by minimising the misfit function:

$$\Phi \;=\; \delta\mathbf{d}^{\dagger}\mathbf{W_d}^{\dagger}\mathbf{W_d}\delta\mathbf{d} \tag{3}$$

with $\delta\mathbf{d} = \mathbf{d_{obs}} - \mathbf{d_{cal}}$ the data residual vector. In CSEM inversion the data vectors usually contains each component of the electric and/or magnetic fields for each station, source and frequencies. However, other kind of observable can be used to build the data vector. In the framework of local linear inversion, we want at each iteration to determine the model update $\delta\mathbf{m}$ solution of the Gauss-Newton equation:

$$\Re(\mathbf{J}^{\dagger}\mathbf{J})\delta\mathbf{m} \;=\; -\Re(\mathbf{J}^{\dagger}\mathbf{W_d}\delta\mathbf{d}) \tag{4}$$

where $\mathbf{J}$ is the sensitivity matrix.

Reparameterization of the problem

The sensitiviy $\mathbf{J}$ decreases rapidly with the distance from the source, resulting in a very poorly conditioned linear system to be solved. In POLYEM3D, this linear system is solved with LSQR that is known to be efficient for poorly conditioned linear system. Preconditionning is also applied by model reparameterization. Instead of performing inversion of ρ, we can inverse $\mathbf{m}$:

$$\mathbf{m} \;=\; \mathbf{G}^{-1}\mathbf{D}^{-1}C(\rho) \tag{5}$$

with C a change of variable (such as logarithm), D an arbitrary linear operator that rescale the sensitivity loss with depth (Plessix and Mulder, 2008; Bretaudeau et al., 2016), and G a linear operator that change the basis of description of the model (for instance a basis of splines described on a coarse grid). Each line of the sensitivity matrix thus can be written:

$$\mathbf{J}^{\mathbf{c}}_{\mathbf{s},\mathbf{r}} = \mathbf{G}^t\mathbf{D}^t \frac{1}{C'(\rho)} \mathbf{E_s} \frac{\partial \mathbf{A}}{\partial \rho} \mathbf{A}^{-1} \wp^{ct}_r \tag{6}$$

A pseudo-MT formulation

The reparameterization allows to perform efficient 3D inversion for MT or multiple source CSEM. However it is still not enough to inverse CSEM data when a single source is used, as the sensitivity singularity at the source cumulates over each line of **J**. We found that recasting the data acquired with two different transmiters using a MT tensor formulation mitigates the singularity due to the transmiter both in the data and in the sensitivities (Bretaudeau et al., 2016). Taking for a station the definition of a Z tensor as a transfer function:

$$\begin{pmatrix} E^x \\ E^y \end{pmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{pmatrix} H^x \\ H^y \end{pmatrix}, \tag{7}$$

and considering two different sources (it can be typically two polarization of a single transmiter), we can obtain for each station the 4 components of this pseudo-MT tensor by a combination of the 8 electric and magnetic fields generated by those two sources:

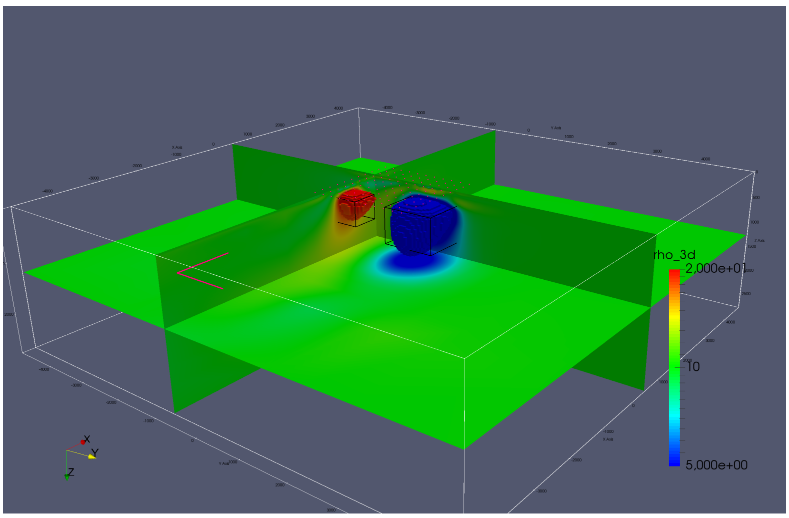
$$Z_{ij} = f(E^c_s, H^c_s). \tag{8}$$

Recasting the CSEM data under this form reduce the number of data by 2 and results in a sensitivity matrix that is a linear combination of the common CSEM sensitivities weighted by the values of the fields:

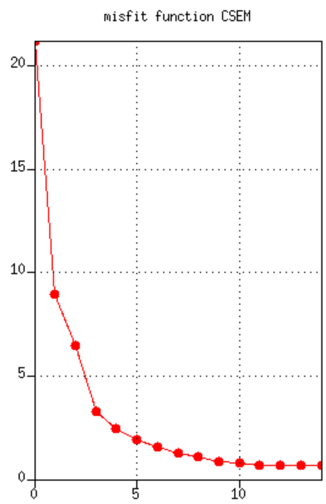
$$J_{Z_{ij}} = f(J_{E^c_s}, J_{H^c_s}, E^c_s, E^c_s) \tag{9}$$

The pseudo-MT tensor is not to be linked to an apparent resistivity or a MT tensor because depending on the frequency and the source-receiver distance considered, the far field condition is not always respected. It is however a well balanced observable that can be inverted if an accurate modeling of the real transmiters is considered.

A toy example



(a) Final resistivity model



(b) Misfit function decrease

Figure 1 *3D resistivity model obtained by frequency-domain CSEM data inversion, using 121 stations, 2 source polarisations and 6 frequencies from 0.5 to 128Hz.*

We illustrate the behavior of the new formulation on a 3D toy synthetic example. The survey is composed of 121 stations over a $10\Omega.m$ medium with 2 anomalies at $1\Omega.m$, and $100\Omega.m$. We consider the inversion of near field CSEM data generated with two orthogonal polarization located at 2km from the closest station, and 6 frequencies from 0.5 to 128Hz. The CSEM data are reformulated using the pseudo-MTtensor formulation.

We show figure 1 the final inversion result. The framework presented previously is used and include Gauss-Newton optimisation, model parameterisation using smooth basis functions, depth preconditionning and data reformulated using pseudo MT tensors. The modeling grid is The inversion was run on 5 nodes with 24 cores each. 15 iterations where completed in about 15 hours. The misfit decrease is shown figure 1b.

The footprint of the transmiter in the sensitivity is not completly removed but is reduced enough to allow convergence of the inversion, and finaly is almost completly removed in the final model. The station array is here in far field condition for the highest frequencies and in near field for the lowest. The deep resistive anomaly in the final model is underestimated and a few artefacts appears around the anomalies but the whole structure is quite well reconstructed and the conductive is very well imaged. Sharper results with less artefacts might however be obtained with more efficient regularization.

Real data example

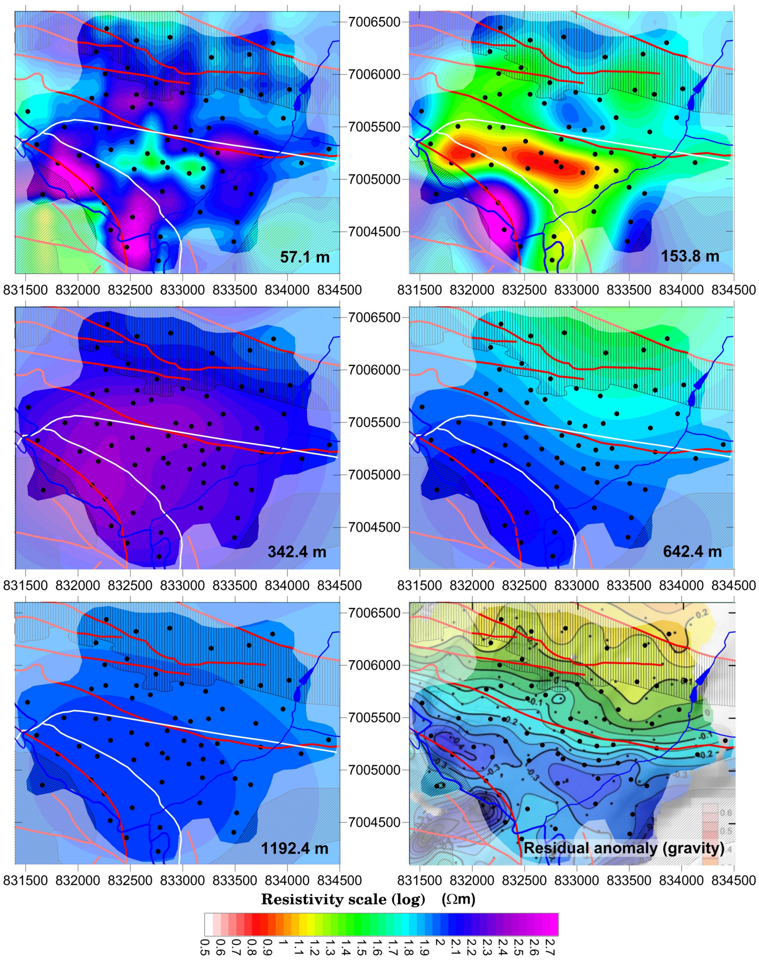


***Figure 2** Slices of the 3D resistivity model and (f) gravity map, with contours from the geological map.*

The same inversion workflow has been applied to a land CSEM survey acquired with a single transmiter in an area where the geology is quite complex with faults and strong 3D effects. The acquisition was performed using 60 stations around the village of Givet in France, close from the Belgium border. The

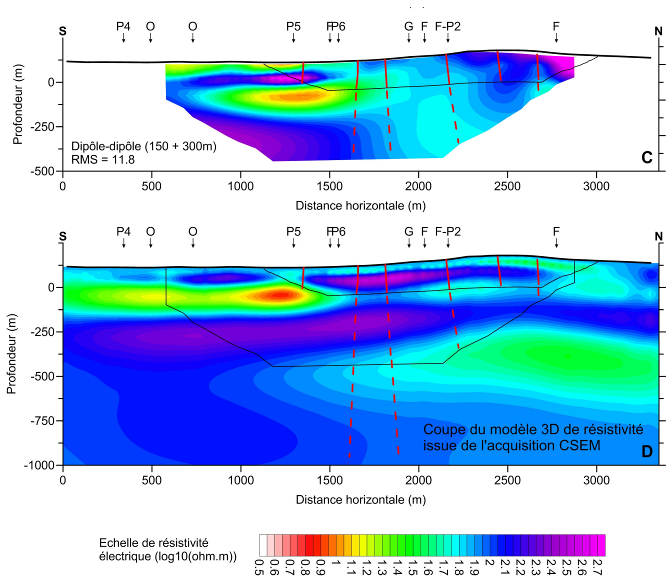


Figure 3 *Cross section extracted from the 3D resistivity model and 2D long DC resistivity profile.*

survey is located at 5 km from a Nuclear power plant, and high level of cultural noise prevent from the use of MT. The target was the intersection of the top of a deeping limestone layer with a fault zone at an expected depth of about 800m. We used for the inversion both electric and magnetic fields recorded from 0.125Hz to 1024Hz. The figure 2 shows slices at different depths. The image is in good accordance with geological knowledges, and in good accordance with the gravity map collected on the same area (figure 2-f). The result is also very well correlated with a long DC electric profile acquired on the same area, as represented figure 3. The shape of the limestones are well defined at the surface and the interaction of the most conductive layer with the regional fault well delineated and estimated at a depth of about 300m.

Conclusions

We proposed a workflow for 3D inversion of land CSEM data. For that we combine Gauss-Newton optimisation with smooth model paramerisation, depth preconditionning and by recasting data as a pseudo-MT tensor. We show on both a synthetic case and an experimental dataset that this formulation allows to perform 3D inversion of CSEM land data using a single transmiter where common approach fails.

Acknowledgements

The research leading to these results has received funding from the EC Seventh Framework Program under agreement No. 608553 (IMAGE). Large scale modeling and inversion was performed on the SRV185 BRGM supercomputer and thanks to GENCI (Grand Equipement National de Calcul Intensif) and CINES (Centre Informatique National de lEnseignement Supérieur) supercomputing facilities. All modelisations and inversions were performed using the POLYEM3D software developped at BRGM in the framework of the IMAGE project.

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