



HAL
open science

Semi-distributed lumped model of a karst system under active management

Bernard Ladouche, Jean-Christophe Maréchal, Nathalie Dörfliger

► To cite this version:

Bernard Ladouche, Jean-Christophe Maréchal, Nathalie Dörfliger. Semi-distributed lumped model of a karst system under active management. *Journal of Hydrology*, 2014, 509, pp.215-230. 10.1016/j.jhydrol.2013.11.017 . hal-00905925

HAL Id: hal-00905925

<https://brgm.hal.science/hal-00905925v1>

Submitted on 18 Nov 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 **Semi-distributed lumped model of a karst system under active** 2 **management**

3 Bernard LADOUCHE^{1*}, Jean-Christophe MARECHAL¹ and Nathalie DORFLIGER²

4 1. BRGM, Water Environment & Ecotechnologies Division, 1039 rue de Pinville, F-34000, France
5 Montpellier, France. <http://www.brgm.fr>; b.ladouche@brgm.fr; jc.marechal@brgm.fr

6 2. BRGM, Water Environment & Ecotechnologies Division, 3 avenue Claude Guillemin, BP 36009
7 F-45060 Orléans cedex 2, France. n.dorfliger@brgm.fr

8 *corresponding author: Tel.: 334 67 15 76 80; Fax: 334 67 15 79 75

9 **Abstract**

10 In this paper inverse modeling is used to characterize the regime of a karst aquifer subjected to
11 extensive pumping in a conduit located upstream of its main outlet. The systemic approach uses a
12 transfer model that is based on computing the convolution integral of up to several signals, e.g.,
13 efficient rainfall, pumping, to simulate flow rates and groundwater levels in both the karst conduit
14 and the carbonate matrix at the aquifer outlet and in several parts of the catchment area. The model
15 is a semi-distributed lumped model which simulates the hydrological response of a heterogeneous
16 karst aquifer made up of different hydrologic compartments, and is applied to the Lez karst system,
17 France. Groundwater is abstracted near the system's major outlet at a higher rate than the low-
18 water spring discharge, thereby mobilizing stored groundwater during low-water periods ('active
19 management').

20 The model's results are very satisfactory, especially for the karst system outlet, where the water
21 levels are particularly well reproduced. The model can also simulate the natural, i.e., non-pumping,

22 state of the karst system and thereby estimate the impact of active management on the water
23 resource.

24 **Highlights**

25 Characterization of karst functioning using a lumped semi-distributed approach

26 Interpretation of a very long times series of groundwater levels in a karst system

27 New insights on the mobilization of stored reserves by pumping within a karst system

28

29 **Keywords**

30 Impulse response

31 Groundwater flow modeling

32 Groundwater level fluctuations

33 Pumping

34 **Introduction**

35 Numerical models for karst aquifers usually fall within two main categories (Goldscheider and Drew
36 2007): lumped-parameter or reservoir models (Dreiss 1983; Pinault et al., 2001a, b, 2004) and
37 distributed physical models (Birk et al., 2003; Liedl et al., 2003). Owing to the high degree of
38 heterogeneity of karst aquifers, distributed models (Birk et al., 2000; Teutsch and Sauter, 1998) can
39 be relatively difficult to implement and calibrate because of difficulties in obtaining the necessary
40 input data. These difficulties have limited the use of such models in karst hydrogeology. Dual-
41 porosity models with implementation of discrete pipe networks within a limestone matrix (Cornaton

42 and Perrochet, 2002) represent a promising approach for testing hypotheses on the conceptual
43 representation of water interactions between the matrix and pipe networks. However, because the
44 available data on karst conduit geometry are generally very limited, this modeling approach is
45 difficult to implement. The lumped-parameter model is simpler and considers the karst as a whole; it
46 uses the relationship between rainfall and discharge (bivariate analysis) to characterize the karst flow
47 regime, and can potentially be used for a karst hydrosystem in which these data are available (Denic-
48 Jukic and Jukic 2003; Dorfliger et al., 2009; Padilla and Pulido-Bosch 2008; Jukic and Denic-Jukic 2009;
49 Martínez-Santos and Andreu 2010, Moussu et al., 2011, Long and Mahler, 2013). However, some
50 karst aquifers are characterized by geographical or hydrogeological subsystems that can be
51 considered as homogeneous units at the scale of interest which potentially are hydraulically
52 connected to each other at the scale of the karst catchment area. In such cases, a lumped parameter
53 model applied to the overall catchment cannot take this hydrodynamic heterogeneity into account.
54 One option, if the necessary data are available, is to consider a lumped parameter modeling
55 approach for each subsystem and then to aggregate the results in order to represent the overall flow
56 regime of the karst aquifer at its outlet. This is an intermediate approach between the lumped
57 parameter model and the distributed physical model, and here is termed a semi-distributed lumped
58 parameter model.

59 Karst aquifers can form major groundwater reservoirs (Bakalowicz 2005) and are locally pumped to
60 meet growing drinking-water requirements due, in particular, to population growth. 'Active
61 management' through pumping from the main karst conduit in the vicinity of its outlet enables
62 optimal exploitation of a karst groundwater by avoiding the negative effects of large low-water
63 discharge variations at the springs. Active management can thus be defined as pumping at a flow
64 rate greater than the spring's low-water discharge rate so as to mobilize the aquifer's stored reserves
65 (Avias 1995). These reserves are then replenished during the following rainy season, resulting in less
66 intense floods at the start of the season (Fleury et al. 2008). In such cases the karst system is

67 characterized by two distinct types of flow regime: (i) a high-water flow regime when the natural
68 outlet discharge is greater than the pumping rate and the system functions naturally; and (ii) a low-
69 water flow regime when the natural outlet discharge is lower than the pumping rate, and the
70 pumped flow rate is the sum of the natural outlet discharge plus the mobilized karst storage flow
71 desaturating the conduits and the matrix (Maréchal et al., 2008). Generally, when a karst system has
72 been exploited over a long period, no observations of the system's natural flow regime are available
73 and it is therefore difficult to reconstruct the system's 'natural' rainfall–discharge relationship: this is
74 a similar to attempting to determine hydrological processes in ungauged basins (Sivapalan 2003).

75 The Lez spring karst system, in southern France, has been an ideal site since the 1970s for studying
76 and modeling a flow regime. Most of the numerical models developed for studying the aquifer's
77 hydrodynamics with a view to groundwater exploitation are based on the laws for draining several
78 reservoirs (Guilbot 1975; Thiery et al., 1983, Fleury et al., 2008). These models are thus unable to
79 describe the functioning of the various hydrogeological units of the hydrosystem, and especially that
80 of the western compartment of the Matelles Fault (Massili 2011; Massili et al, 2011). The main aim of
81 our work here was to develop a lumped parameter modeling approach suitable for (i) a karst system
82 whose overall flow regime results from the juxtaposition of identified subsystems, and (ii) a system
83 that was under 'active management' for a long period of time and for which the natural flow regime,
84 in particular at the outlet, is unknown. The final objective of this work was to develop a modeling
85 approach that is able to predict the water levels and spring discharge so that the impact of global
86 change scenarios on the Lez karst system could be assessed.

87 **Study area**

88 The Lez karst system forms part of the North Montpellieran Garrigue hydrogeological unit delimited
89 by the Hérault river (to the west) and the Vidourle river (to the north and east, Figure 1). The Lez
90 karst aquifer, between 650 and 800 m thick, lies within the Upper Jurassic layers on both sides of the

91 Matelles-Corconne Fault (here termed the 'Matelles Fault'). The aquifer is unconfined in the western
92 compartment of this fault and locally confined in the eastern compartment (Figure 2a). The base and
93 top of the Lez karst aquifer in the eastern compartment are respectively Callovian–Oxfordian marls
94 (Middle Jurassic) and a thick succession of Lower Valanginian (Lower Cretaceous) marls and marly
95 limestone; the aquifer itself straddles in the Upper Jurassic and the Berriasian (Lower Cretaceous).
96 The karstification of the main conduit below its top wall in the eastern compartment was confirmed
97 as -10 m (a.s.l.) by diving, and down to -48 m (a.s.l.) by drilling (Figure 3). The geometry of the
98 conduit is known only between the spring and a few boreholes located 400 meters away: this lack of
99 knowledge at the watershed scale prohibits the application of any distributed modeling approach. A
100 southward structuring of the aquifer, with the development of karstification at depth and an
101 underground drainage in the direction of the Mediterranean, is probable and almost certainly related
102 to the Messinian salinity crisis (Clauzon et al., 2005; Avias 1992). However, the compressive tectonics
103 of the Eocene Pyrenean–Provençal phase, in addition to filling the paleo-outlet with impermeable
104 Miocene and Pliocene sediments from the Messinian canyons, gave this large karst aquifer a
105 'confined karst' character with the presence of upstream overflow springs, such as the Lez spring and
106 the intermittent Lirou spring.

107 The Lez spring with its overflow at 65 m (a.s.l.) is the main outlet from the karst system. The area of
108 the hydrogeological catchment is estimated to be 380 km², based on the geology, identification of
109 the impervious structural limits, dye tracings, and groundwater level dynamics in the network of
110 observation boreholes during the drawdown in the spring's conduit (Thiery et al., 1983, Figure 1).
111 Several recharge zones can be identified within this basin according to the nature of the exposed
112 geological formation: (i) the Jurassic limestone outcrop (between 80 and 100 km²) is the main area of
113 recharge from effective rainfall (Figure 1); (ii) the Cretaceous marly-limestone formations (120 km²),
114 which are much less permeable than the Jurassic, provide leakage to the underlying Jurassic and
115 contain identified loss zones with direct recharge to the underlying Jurassic aquifer from surface

116 streams (swallow holes) and in particular along tectonic faults; (iii) the Tertiary formations (160 km²),
117 which are generally impervious and which contribute insignificantly to the karst aquifer recharge.

118 The Lez spring has been supplying the city of Montpellier since 1854. Prior to 1968, the resource was
119 exploited through spring overflow collection, which ranged between 25 and 600 l/s (Paloc 1979).
120 From 1968 to 1982, abstraction was carried out by pumping in the panhole at a rate of some 800 l/s.
121 This period was the prelude to 'active management' of the Lez karst system, with pumping rates in
122 the summer periods being greater than the natural discharge without any pumping. By the end of the
123 1970s increased demand for drinking-water supply had motivated the managers to extract the
124 resource at a rate higher than 800 l/s. For this reason, in 1982 deep wells were drilled into the main
125 karst conduit upstream of the spring in order to obtain a maximum yield of 2000 l/s (Avias 1995). The
126 current pumped rates at the low-water stage (between 1200 and 1700 l/s) are greater than those
127 pumped at high water (900 l/s). The maximum pumping rate is now fixed at 1700 l/s so that the
128 minimum water level (Figure 4) is no lower than 35 m.a.s.l., i.e., 30 m below the elevation of the
129 spring's outlet. A reserve flow of 160 l/s returns to the Lez River downstream of the spring when the
130 spring is not overflowing.

131 The karst system's natural discharge (Q_n) is unknown, because the Lez spring has been tapped since
132 1854 (Paloc 1979). However, the pumping rate (Q_p) has been measured since 1974 and the spring's
133 residual overflow discharges (Q_r) were reliably measured between 1987 and 2007. The natural high-
134 water flows over this period can be estimated by the equation $Q_n = Q_r + Q_p$ (Figure 3a) —the natural
135 discharges (Q_n) estimated from these measurements are shown in Figure 5a.

136 At low-water stage (Figure 3b), the pumping rate (Q_p) is between 1200 and 1700 l/s, which is higher
137 than the natural discharge (Q_n) would have been in the absence of pumping. The pumping caused
138 the spring to dry up ($Q_r = 0$). Accordingly, the pumped discharge (Q_p) during the low-water period is
139 interpreted as the sum of the flow that would have occurred naturally in the absence of pumping

140 (Q_n) and the storage flow (Q_s) mobilized by the pumping: $Q_p = Q_n + Q_s$. The yield obtained from the
141 karst storage (Q_s) is interpreted as coming from depletion of the karst conduits and mobilization of
142 matrix water from the carbonates in which the karst drainage network has been developed
143 (Maréchal et al, 2008). Pumping thus mobilizes the karst system's water reserves that are barely or
144 not at all accessible under the karst's natural flow regime. This is reflected in a lowering of the
145 piezometric head (H_4) in the karst conduit ($35 \text{ m} < h_4 < 65 \text{ m a.s.l.}$; Figure 3b).

146 The Suquet borehole (Figure 2a) intersects the karst conduit connected to the Lirou spring, which is
147 the main outlet from the Western compartment of the Lez karst system (Paloc 1987): it thus enables
148 a description of the overall hydrodynamics of this part of the system. During high-water periods
149 ($H_2 > 65 \text{ m.a.s.l.}$) the groundwater-level response to effective rainfall recharge is very similar to that
150 recorded at the Claret observation borehole. This is located in a conduit in the immediate vicinity of
151 the Matelles Fault (Fig. 1), which is one of the main drainage axes of the karst system (Lacas 1976;
152 Drogue and Grillot 1976; Bérard 1983; Karam 1989). During low-water periods ($h_2 < 65 \text{ m.a.s.l.}$),
153 groundwater-level monitoring reveals the existence of a 'groundwater disconnection' between the
154 two aquifer compartments located on either side of the Matelles Fault (Figure 4). The groundwater
155 level in the western compartment (Suquet borehole) is no longer influenced by drawdowns caused
156 by pumping when the groundwater head of the karst conduit at Claret is 53.5 m.a.s.l. or 52 m.a.s.l. at
157 the Lez observation borehole (Figure 4). The measured groundwater levels at the Suquet observation
158 borehole can be explained if one considers that a 'cascade'-type disconnection exists in the conduit.
159 This does not preclude a water flow from the western compartment continuing to supply the Jurassic
160 located below the Cretaceous cover in the eastern compartment, but the two hydraulic heads
161 measured on either side of the Matelles Fault are independent. The Suquet observation borehole is
162 representative of the karst system draining the western compartment of the Matelles Fault.

163 The groundwater level H_3 measured in the Laudou observation borehole (Figure 1) also appears to
164 be influenced by the pumping (Conroux 2007); but indicates a different hydrodynamic response from

165 the conduit (Lez, Claret, Figure 4), which exhibits a much more inertial groundwater behavior
166 (Conroux, 2007), one that we assume is representative of the matrix storage beneath the Cretaceous
167 cover to the east of the Matelles Fault (Figure 2a).

168 **Semi-distributed lumped model**

169 The semi-distributed lumped modeling approach developed for the present study is based on the
170 state of knowledge concerning the Lez karst system. The architecture of the hydrological model is
171 shown in Figure 2b. Four sub-models (or modules) was used to reproduce the hydrodynamic flow
172 regime of a complex karst aquifer, composed of two compartments (the western and eastern parts
173 of the Matelles fault), and subjected to an ‘active management’ through pumping at the Lez spring
174 (Figure 2a). The first three modules simulate (i) the natural discharge of the Lez spring (Q_n), and the
175 water levels in (ii) the karst conduit of the western compartment at the Suquet observation borehole
176 (ΔH_2) and (iii) the matrix of the eastern compartment at the Laudou observation borehole (ΔH_3). The
177 fourth module calculates the groundwater level of the main karst conduit at the Lez spring (ΔH_4)
178 from the contributions (outputs) of the three intermediate transfer modules.

179 The hydrogeologic modeling was performed using transfer methods implemented with TEMPO
180 software (Pinault, 2001). The calculation of the impulse responses of the transfer modules is
181 achieved by inverse modeling with stress positivity. The term inverse modeling is here restricted to
182 the parameter identification of convolution kernels used to simulate the output from the input.
183 Computation of the impulse response uses regularization methods inspired by Tikonov and Arsenime
184 (1976), Tikonov and Goncharsky (1987), and Dietrich and Chapman (1993), as described by Pinault et
185 al. (2001a). The calculation of impulse responses under stress allows to satisfying the principle of
186 conservation of the water mass. The impulse responses can be parametric or non-parametric. In the
187 case of a non-parametric impulse response, the system is termed ill-posed from a numerical point of
188 view because it has an infinite number of solutions. As mentioned by Doherty and Skahill (2006), the
189 so-called “Tikhov regularization”, in which the parameter estimation process is formulated as a

190 constrained minimization problem, enables an efficient and stable solution to the inverse problem to
 191 be obtained. The mathematical and numerical bases used in Tempo software for the calculation of
 192 impulse responses are presented in Pinault et al. (2001a). The search for a solution by the
 193 regularization technique seeks to reduce the number of degrees of freedom of the optimization
 194 problem by minimizing the norm of the transport equation (i.e eq. 7, eq. 9, eq. 11 and eq. 13 in this
 195 study). Among all possible solutions, the calculation of minimizing the norm of the solution aims to
 196 select the one with the smallest norm in order to smooth the non-parametric impulse response. As
 197 indicated by Pinault et al. (2001a), the regularization parameter (w) is optimized by using the method
 198 of the curve L. This technique takes its name from the shape of the curve obtained when the x-axis
 199 represents the norm of the solution and the y-axis the difference between the model and the
 200 observations. The regularization parameter (w) is selected as close as possible to the angular points
 201 of the curve L. This choice is a compromise between (1) choosing a very smooth solution which
 202 produces a poor fit, and (2) choosing a very irregular solution producing an excellent fit between the
 203 model and the observations. The regularization parameter (w) is the absolute value of the derivative
 204 of the curve L. The second parameter of the non-parametric impulse response is the maximum
 205 length over which the computation is performed.

206 The modeling approach using Tempo software enables a characterization of the flow regime of
 207 complex hydrosystems such as those found in karst systems (Pinault et al., 2001b, 2004; Dörfliger et
 208 al., 2009) as well as in more homogeneous porous media (Pinault et al. 2005; Pinault and
 209 Schomburgk 2006; Pinault and Allier 2007).

210 **Effective rainfall**

211 The modules use the same daily input data to compute the effective rainfall, with the possibility of
 212 using several rain gauges (p being the number of rain gauges) for the rainfall data:

$$213 \quad R_{\Sigma} = \sum_{k=1,p} \delta_k R_k \quad \delta_k \geq 0 \quad k = 1, \dots, p \quad \sum_{k=1,p} \delta_k = 1$$

214 The sum R_{Σ} of a series of rainfall data is defined so as to optimize the contribution of each rain gauge
 215 to the transfer model of the spring's discharge (Module 1). The principle, given by Pinault et al.
 216 (2005) consists of computing the weighting factors (δk) of the rain gauges so that they maximize the
 217 cross-correlation between rainfall and system output.

218 The amount of effective rainfall $R_{eff}(t)$ inducing a flow variation for a time t is calculated by:

$$219 \quad R_{eff}(t) = R_{\Sigma}(t) - \Omega(t) \text{ if } R_{\Sigma}(t) \geq \Omega(t) \text{ otherwise } R_{eff}(t) = 0 \quad \text{Eq. 1}$$

220 The effective rainfall (R_{eff}) is calculated by computing the threshold function Omega:

$$221 \quad \Omega(t) = Gr * R_{\Sigma}(t) + GTa * Ta(t) + C \quad \text{Eq. 2}$$

222 where $*$ is the discrete convolution product, t the time, C a constant, and Gr and GTa the impulse
 223 responses such that $GTa > 0$ and $Gr < 0$. These two impulse responses (Gr and GTa) are represented
 224 by trapezia (Figure 5c). Details of the impulse response parameter calculation are presented in
 225 Pinault et al. (2001a).

226 The threshold $\Omega(t)$ is used to express the deficit of the effective reserve in the ground. Rain decreases
 227 the $\Omega(t)$ threshold, following the reservoir's recharge, while air temperature (Ta) increases it. The
 228 convolution of rain with the negative part of the impulse response (Figure 5c) gives the contribution
 229 of rain threshold $\Omega(t)$. The threshold $\Omega(t)$ (Eq. 2) can be computed using the air temperature (Ta)
 230 because the input and output variables are reduced (divided by their mean). Consequently, only the
 231 variations are involved and it is these that determine the model's accuracy (Pinault 2001). The air-
 232 temperature records used in the calculation (Eq. 2) are from the St-Martin de Londres meteorological
 233 gauging station (Figure 1).

234 The effective rainfall $R_{eff}(t)$ is divided into two components: $R_{eff}(t) = R_s(t) + R_f(t)$, where $R_s(t)$ and $R_f(t)$
 235 are the components of effective rainfall that respectively induce the slow and fast responses of the

236 system. The fast component is assumed to be representative of the fast infiltration which occurs only
237 after high-rainfall events. The component $R_f(t)$ is expressed according to $R_{eff}(t)$ by the Alpha
238 function [$\alpha(t)$]:

$$239 \quad R_f(t) = \alpha(t) \times R_{eff}(t) \quad \text{Eq. 3}$$

240 where $\alpha(t)$ is the proportion of effective recharge (its values lie between 0 and 1). The Alpha function
241 enables consideration of the non-linear switching behavior of karst systems such as 'Flush flow' which
242 depends on the effective rainfall over the preceding days. The Alpha function is related to the effective
243 recharge by the convolution product:

$$244 \quad \alpha(t) = \Gamma_{\alpha, Reff} * R_{eff}(t) \quad \text{Eq. 4}$$

245 where * is the discrete convolution product, t the time, and $\Gamma_{\alpha, Reff}$ the impulse response of $\alpha(t)$ to
246 effective rainfall $R_{eff}(t)$. The impulse responses are represented by trapezia (Figure 5d).

247 **Module 1: Transfer model for the rainfall –Lez spring natural discharge relationship**

248 The first module (Figure 2b) computes the natural discharge of the Lez spring (Q_n) which would flow
249 naturally in the absence of pumping and the mobilized storage flow (Q_s). Inverse modeling of the
250 natural flow consists of calculating both the slow (τ_s) and fast (τ_f) impulse responses associated with
251 the different types of flow that occur within the karst system. The fast transfer (τ_f) may be
252 associated with 'flush flow'. This can occur when hydraulic continuity is established between the
253 unsaturated and saturated zones of the karst through a temporary connection between groundwater
254 in the epikarst and the saturated zone, via fractures and/or subvertical conduits (Pinault et al.,
255 2001b). 'Flush flow' has been demonstrated in the Fontanilles and Cent-Fonts karst systems to the
256 north of Montpellier (Aquilina et al., 2005, 2006) during periods of major recharge. The impulse
257 response of the fast flow (τ_f) can also describe the contribution of losses from temporary streams

258 located in the catchment area of the Lez spring. In the model, the fast transfer of the recharge (τf) is
 259 obtained by the convolution product of a Gaussian function with a decreasing exponential:

$$260 \quad \begin{cases} \tau f(t_i) = A \cdot \exp\left(-\ln(2) \left(\frac{t_i - T}{D}\right)^2\right) * \exp\left(\frac{-t_i \ln(2)}{L}\right) & \text{if } 0 \leq t_i \leq \tau \\ \tau(t_i) = 0 & \text{if } t_i > \tau \end{cases} \quad \text{Eq. 5}$$

261 From a physical viewpoint, the model reflects the recharge of a reservoir (Gaussian function)
 262 followed by its rapid depletion represented by the decreasing exponential law. The succession of
 263 these two phenomena is described by the convolution product (*). The parameter T expresses the
 264 delay of the recharge process with respect to the rainfall, while the parameter D reflects the duration
 265 of the event. A is the normalization constant.

266 The slow transfer (τs) brings into play the various infiltration and depletion processes inducing a
 267 more-or-less significant delay between the rainfall and the observed discharge. The depletion of the
 268 saturated zone is also described by this slow transfer. In the approach we adopted, the slow transfer
 269 of the recharge (τs) is described by a parametric model (Denic-Jukic and Juckic 2003) that convolves
 270 an analytical function consisting of two reservoirs in parallel, and the depletion of the saturated zone
 271 is described by a decreasing exponential function. The choice of this type of parametric model can be
 272 justified by the existence of the two karst compartments located on either side of the Matelles Fault.
 273 The expression for the two unequal parallel linear reservoirs with recession coefficients k_1 and k_2 is:

$$274 \quad \tau s = \frac{\theta}{k_1} e^{-t/k_1} + \frac{1-\theta}{k_2} e^{-t/k_2} \quad \text{Eq. 6}$$

275 where the parameter θ represents the flow ratio between the two reservoirs. The impulse response
 276 is presented on Figure 5e.

277
 278 The discharge $Qn(t)$ is computed by the following equation (Eq. 7):

$$279 \quad Qn(t) = A (\tau f * Rf + \tau s * Rs) \quad \text{Eq. 7}$$

280 Where * represents the discrete convolution product, t the time, $Qn(t)$ the flow of the hydrosystem,
 281 A the recharge area of the catchment basin (where recharge occurs), τf and τs the normalized
 282 impulse responses (the area being the unity), and Rf and Rs the components of the effective rainfall
 283 ($Reff = Rf + Rs$) inducing the fast and slow transfers respectively. The inverse modeling process
 284 calculates the normalized impulse responses τs and τf , the effective rainfall [$R_{eff}(t)$], and the part of
 285 effective rainfall [$\alpha(t)$] that is involved in quick flow.

286 The karst storage flow (Qs) mobilized by the pumping is then calculated as follows:

$$287 \quad \begin{aligned} & \text{if } Qp \geq Qn \text{ then } Qs = Qp - Qn \text{ and } Qr = 0 \\ & \text{if } Qp < Qn \text{ then } Qs = 0 \text{ and } Qr = Qn - Qp \end{aligned} \quad \text{Eq. 8}$$

288 where Qs represents the karst storage flow, Qp the pumping rate, and Qn the natural flow in the
 289 absence of pumping. The karst storage flow (Qs) is negative (by convention) and is assumed to be
 290 zero when there is overflow at the outlet (high water). The karst storage volume Vs calculated from
 291 Qs is different from the dynamic volume Vd which would have flowed in the absence of pumping, the
 292 latter generally being assessed from the analysis of recession curves (Mangin 1975).

293 **Module 2: Transfer model for the rainfall – Lirou spring discharge relationship (Western**
 294 **compartment)**

295 The second or Suquet module (Figure 2b) simulates the hydraulic head of the conduit (ΔH_2) in the
 296 Jurassic compartment west of the Matelles Fault (Suquet borehole; Figure 1 and Figure 2a) near the
 297 Lirou overflow spring. The relationship between the recharge (input) and the piezometry (output) is
 298 based on computing a non-parametric impulse response

$$299 \quad H2(t) = \frac{1}{\eta 2} \Gamma_2 * R_{eff} + \varepsilon \quad \text{Eq. 9}$$

300 where Γ_2 is the normalized impulse response (area equal to 1) of the effective rainfall recharge and
 301 $\eta 2 = Ws \times Max(\tau_2)$, a constant linked to the porosity (Ws) through determination of the base

302 piezometric level H_0 (Pinault 2001) and the maximum value of the normalized impulse response (τ_2);
 303 ε represents the erratic contribution that is not described and explained by the model. The
 304 computation of the impulse responses is performed under positivity constraint and while satisfying
 305 the principle of conservation of the mass of water, the groundwater level being expressed relative to
 306 a reference level H_0 . This reference level is the groundwater level in the absence of external
 307 solicitation (effective recharge, pumping). The inverse modeling is set up to calculate the normalized
 308 impulse response $[I_2]$ (Figure 6c), the threshold $\Omega(t)$ (Figure 6b), and the effective rainfall.
 309 Parameters are obtained through inversion by minimizing the quadratic form:

$$310 \sum_{i=1}^N (H2_{obs}(t_i) - H2_{mod}(t_i))^2 \quad \text{Eq. 10}$$

311 The slow component can reproduce the hydrological response of the western compartment (Figure
 312 6c). The fast component is not necessary for this module.

313 **Module 3: Transfer model for the matrix storage (Eastern compartment)**

314 The third or Laudou module is used to characterize the functioning of the Jurassic karst beneath the
 315 Cretaceous cover (Figure 2b) in the eastern compartment of the Matelles Fault. This transfer model
 316 takes into account the recharge by effective rainfall as well as the effect of the karst storage flow (Q_s)
 317 mobilized by pumping.

318 The piezometric evolution is described by the following convolution equation:

$$319 \quad H3(t) = \frac{1}{\eta^3} \cdot I_3 * R_{eff} + \lambda_{3s} \cdot I_{3s} * Q_s + \varepsilon \quad \text{Eq. 11}$$

320 where I_3 and I_{3s} are the normalized non-parametric impulse responses (areas equal to 1), I_3 is the
 321 normalized impulse response of the recharge from effective rainfall, and I_{3s} is the normalized
 322 impulse response of the mobilized karst storage flow; ε represents the erratic contribution that is not
 323 described and explained by the model. The parameters $\frac{1}{\eta^3}$ and λ_{3s} are positive coefficients for
 324 expressing the conservation of the water mass:

$$325 \quad \frac{1}{\eta_3} \cdot \frac{\overline{R_{eff}}}{\Delta H_3} + \lambda_{3s} \cdot \frac{\overline{Q_S}}{\Delta H_3} = 1 \quad \text{Eq. 12}$$

326 The term $\frac{1}{\eta_3} \cdot \overline{R_{eff}}$ is used to describe the relative contribution of the recharge and $\lambda_{3s} \cdot \overline{Q_S}$ describes
 327 the relative decrease in piezometry resulting from the karst storage flow (Q_S) mobilized by pumping
 328 in the conduit of the Lez spring.

329 The inverse model of fissured matrix storage in the Jurassic karst beneath the Cretaceous cover is
 330 represented in Figure 7. The inverse modeling calculates the threshold $\Omega(t)$ (Figure 7b) and the
 331 effective rainfall, the normalized impulse response of the recharge (Γ_3), and the normalized impulse
 332 response of the karst storage flow mobilized by pumping (Γ_{3s}). The impulse responses of the two
 333 contributions Γ_3 and Γ_{3s} are shown in **Figure 7b**. The computation of the impulse responses is
 334 performed under positivity constraint, the groundwater level being expressed relative to a reference
 335 level H0. The fast component is not necessary for this module.

336 **Module 4: Transfer model for the main conduit to the Lez springhead**

337 Module 4 (Figure 2b) enables a simulation of the water level of the Lez karst conduit in the
 338 immediate vicinity of the spring (borehole located 400 m upstream of the Lez spring) and thus to
 339 characterize the global hydrodynamics of the system at the outlet. Module 4 is a system convolved in
 340 sequence since the outputs of Modules 2 and 3 [$H_2(t)$ and $H_3(t)$] are the inputs to the convolution
 341 kernel of the simulation model for the water level [$H_4(t)$].

342 The piezometry is calculated by the following convolution equation:

$$343 \quad H_4(t) = \lambda_a \cdot \Gamma_a * R_{eff} + \lambda_b \cdot \Gamma_b * Q_S + \lambda_c \cdot \Gamma_c * H_2 + \lambda_d \cdot \Gamma_d * H_3 + \varepsilon \quad \text{Eq. 13}$$

344 where Γ_a , Γ_b , Γ_c and Γ_d are the normalized non-parametric impulse responses (areas equal to 1); Γ_a
 345 is the normalized impulse response of the effective rainfall recharge; Γ_b is the normalized impulse
 346 response of the mobilized karst storage flow (Q_S); Γ_c is the normalized response of the supply

347 component from the Jurassic aquifer lying to the west of the Matelles Fault (computed by the Suquet
 348 module); Γ_d is the normalized response of the supply component from the matrix (computed by the
 349 Laudou module); and ε represents the erratic contribution that is not described or explained by the
 350 model.

351 λ_a , λ_b , λ_c and λ_d are positive coefficients for expressing the conservation of the water mass:

$$352 \quad \lambda_a \cdot \frac{\overline{R_{eff}}}{\overline{\Delta H_4}} + \lambda_b \cdot \frac{\overline{Q_S}}{\overline{\Delta H_4}} + \lambda_c \cdot \frac{\overline{\Delta H_2}}{\overline{\Delta H_4}} + \lambda_d \cdot \frac{\overline{\Delta H_3}}{\overline{\Delta H_4}} = 1 \quad \text{Eq. 14}$$

353 The term $\lambda_b \cdot \overline{Q_S}$ allows us to describe the relative decrease caused by the mobilized karst storage
 354 flow (Q_S). The term $\lambda_c \cdot \overline{\Delta H_2}$ describes the relative contribution of the Jurassic aquifer on the west of
 355 the Matelles Fault. The term $\lambda_d \cdot \overline{\Delta H_3}$ allows us to describe the relative contribution from the matrix
 356 reservoir. In this model, the term $\lambda_a \cdot \overline{R_{eff}}$ does not allow a description of the contribution from the
 357 effective rainfall; we fixed this parameter at 1% because we consider that most of the effective
 358 rainfall recharge information is already contained in the model's other components
 359 (ΔH_2 and ΔH_3). This component enables one to describe the recharge phenomena associated with
 360 the fast flow that occurs during significant flooding ($Q > 3000$ l/s). We assume that the component
 361 described refers to direct infiltration from losses associated with the temporary streams that form
 362 during floods.

363 **Results**

364 Rainfall data from three weather stations were used (Fig. 1): Saint-Martin de Londres (R1),
 365 Montpellier-Fréjorgues (R2), and Valflaunes (R3). The aim of the computation was to maximize the
 366 cross-correlation between R_Σ and the overflow discharge (Q_r) at the Lez spring measured between
 367 1987 and 2007. The linear combination obtained for R_Σ is:

$$368 \quad R_\Sigma = 0.34 \cdot R1 + 0.53 \cdot R2 + 0.13 \cdot R3 \quad \text{Eq. 14}$$

369 The calibration of Module 1 was based on the natural high-water discharges ($Q_n = Q_r + Q_p$ for the
370 period 1998-2006; Figure 5a); the natural low-water discharges are not known. The parameters of
371 the transfer function (Eq. 6) shown on Figure 5e are: $k_1 = 7.4$ days, $k_2 = 59$ days, $\theta = 0.29$. The
372 length of the impulse response is 324 days. The mean transit time (T_m) associated with the
373 normalized impulse response is the mean value of the lag (Pinault et al, 2005). The average transit
374 time (T_m) is 42 days. The first transfer model assumes that the regulation time (system memory) of
375 the slow component of the karst system is less than a year (324 days; Figure 5e). This assumption is
376 not restrictive because the karst system's spring begins to flow again with the first autumnal rains,
377 and the low-water periods (recession) do not exceed a year. The model was validated with the
378 natural high-water discharges for the 1987–1997 period (Figure 5b); it satisfactorily reproduces the
379 high-water periods, and the Nash criterion of 0.67 (Nash and Stutcliff, 1970) is considered
380 satisfactory, having been calculated on the high-water cycles for the 1987–2007 period. The area of
381 the karst catchment basin providing recharge is estimated at 116 km^2 . The fast component (Q_f) of
382 the recharge flows only during significant floods ($Q_n > 3000 \text{ l/s}$) and contributes up to a maximum 7%
383 of the flood discharge.

384 For Module 2, the reference level is that measured at low water when the head becomes constant
385 ($H_0 = 60 \text{ m.a.s.l.}$) following groundwater-level disconnection from the Jurassic compartment lying east
386 of the Matelles Fault. The groundwater level is described only by the slow component of the
387 recharge, the fast component being unnecessary for the simulation. The impulse response for the
388 slow component is shown in Figure 6c. The average transit time (T_m) of the normalized impulse
389 response (τ_2) is 42 days. The regularization parameter (w) is 0.1. The length of the impulse response
390 is 96 days. Module 2 enables a satisfactory reproduction of the daily changes of the groundwater
391 level measured at the Suquet observation borehole (high Nash criterion = 0.88). The model was
392 validated with the water levels measured for the 2011–2012 period (Figure 6c): it satisfactorily
393 reproduces the high-water and low-water periods, and the Nash criterion of 0.82 is considered

394 satisfactory. The model nevertheless tends to overestimate the groundwater level after the first
395 floods of the new hydrological cycle (autumn). The effective porosity (η_2) is estimated at 0.37%,
396 which is a general value for the aquifer compartment drained by the Lirou overflow spring; the
397 Suquet observation borehole taps the same drainage system just upstream of the intermittent
398 spring. This second transfer model was used to simulate the groundwater level of the western
399 compartment (1974–2010; Figure 9e) and the output of this module 2 modeling is one of the inputs
400 to the fourth module of the Lez transfer model.

401 In Module 3, computing the effective porosity ($w_s = \eta_3 / \text{Max}[\Gamma]$) requires a knowledge of the base
402 level (H_0) corresponding to the piezometric level reached by the system in the absence of recharge
403 and pumping (Pinault 2001, Pinault et al., 2005). Because in the present case the base level (H_0)
404 cannot be deduced from the observed piezometric record, owing to the influence of pumping, it was
405 characterized using the model using trial and error to obtain a maximum Nash criterion. The base
406 level obtained is $H_0 = 80$ m.a.s.l., which is an intermediate value in the observed variation range (61–
407 115 m.a.s.l.). The effective porosity (η_3) is estimated at 0.91%. The average transit times (T_m) of the
408 normalized impulse responses of the two contributions (Γ_3 and Γ_{3s}) are 47 days and 9 days
409 respectively (Figure 7b). The regularization parameter (w) is 0.1. The length of the impulse response
410 is 160 days. For offsets longer than 50 days, the non-parametric impulse response of effective rainfall
411 shows oscillations which correspond to noise without physical meaning. Nevertheless, the length of
412 the impulse response is significant because the tests conducted with a shorter length for the impulse
413 response do not allow the modeling of the inertial response of the aquifer to effective rainfall. This
414 third transfer model provides a satisfactory simulation of the daily evolution of the measured
415 piezometry for the calibration period (Nash criterion = 0.82, Figure 7a). The model was validated with
416 the water levels measured for the 2008–2011 period (Figure 5d); it satisfactorily reproduces the high-
417 water periods, and the Nash criterion of 0.69 is considered acceptable. The average recharge
418 contribution is estimated at about 71.5%, with the mobilized karst storage flow (Q_s) explaining on

419 average 28.5% of the measured piezometric variations. When the water level drops below the
420 reference level ($H_0 = 80$ m), the piezometric variation appears to be controlled mainly by the
421 mobilized storage flow (Q_s) component, which can thus cause drawdown in the piezometer of nearly
422 13 m. This third model was used to estimate the groundwater level within the karst for the period
423 before 2003 and for that after 2005. The record produced by this modeling forms one of the inputs
424 to the fourth module of the Lez transfer model (1974–2008; Figure 9e).

425 The impulse responses (Γ_a , Γ_b , Γ_c and Γ_d) of Module 4 were computed for the shortest possible lags
426 (4 days, Figure 8c) to make the model stable, since the $H_2(t)$ and $H_3(t)$ variables are strongly
427 correlated (Figure 10). The average transit times (T_m) of the normalized impulse responses of the
428 two contributions Γ_a and Γ_b are 2.5 days and 2 days respectively, but are less than 1 day (0.9 day) for
429 contributions Γ_c and Γ_d . The regularization parameter (w) is 0.1 and the length of the impulse
430 response is 4 days. The reference level H_0 , estimated by trial and error so that the model would be as
431 close as possible to the observed groundwater level, is $H_0 = 59.9$ m.a.s.l. Note that this reference level
432 is similar to that obtained by the Suquet method ($H_0 = 60$ m) which reflects the moment when the
433 karst west of the Matelles Fault is piezometrically disconnected from the rest of the aquifer located
434 beneath the Cretaceous cover.

435 The model gives a very satisfactory reproduction of the daily changes of the measured groundwater
436 level (calibration period from 29/04/2002 to 25/02/2004) with a Nash criterion of 0.97 (Figure 8a)
437 and a very low standard deviation (0.02). The model has been used to simulate the groundwater
438 level of the karst conduit for the periods before 2002 and after 2004 (Figure 9e). The model was
439 validated with the water levels measured before 2002 (1974–2002 period) and after 2004 (2004-
440 2011 period) (Figure 5a). This transfer model gives a satisfactory simulation of the daily changes of
441 the water level measured in the conduit (Nash criterion = 0.68). The periods of aquifer drawdown
442 corresponding to the spring drying up are well reproduced. The simulated flood water levels are in
443 some cases greater than the measured levels.

444 Most of the groundwater level variation (44%, Figure 9b) relative to the reference level ($H_0=59.9$
445 m.a.s.l.) appears to be explained by the impulse response of the mobilized karst storage flow (Q_s).
446 The average contribution of the matrix reservoir to the groundwater-level variation in the karst
447 conduit is around 36%. The remaining part (20%) comes from the contribution of the karst lying west
448 of the Matelles Fault. When the groundwater level in the conduit falls below 59.9 m.a.s.l. (reference
449 level), the groundwater level variations appear to be mainly controlled by the mobilized karst storage
450 flow (Q_s), whose average contribution becomes greater than 90%.

451 **Discussion**

452 **Sensitivity**

453 The time-invariant impulse response (IR) used to describe the dynamic response of a system to an
454 instantaneous impulse can be objectively chosen as a mathematical function selected on physical
455 grounds, or by the use of a non-parametric approach (Von Asmuth et al, 2002). In this study, the
456 choice focused on the non-parametric approach because we believe that karst processes are not well
457 explained by physical laws only. Indeed, the shape of the impulse response to an instantaneous
458 impulse of recharge is the result of diffuse, fracture, and conduit flows with the potential existence of
459 turbulent flows, themselves conditioned by vertical and horizontal heterogeneities. In order to assess
460 the model's sensitivity to the choice of the non-parametric impulse response, rather than a
461 parametric one, we also calibrated the transfer module of Suquet borehole with a parametric model.
462 The convolution function used to perform the sensitivity test is a Gaussian-Exponential function (Eq.
463 5). Using this model, the recharge of a reservoir is described by the Gaussian function
464 $[\exp(-\ln(2) \left(\frac{t_i - T}{D}\right)^2)]$ whereas the decreasing exponential function $[\exp(-\frac{t_i \ln(2)}{L})]$ allows to
465 represent the drainage from the reservoir; the succession of these two phenomena is the
466 convolution product of the recharge law by the discharge law. The normalized impulse response of
467 the parametric model is reported in Figure 6c, enabling a comparison with the impulse response of

468 the non-parametric model. The non-parametric model produces a better fit than the parametric
469 model both for the calibration period (Nash= 0.88 and 0.82, respectively) and for the validation
470 period (Nash= 0.82 and 0.69, respectively, Figures 6b and 6d). The main difference concerns the peak
471 response to the effective recharge, which occurs after a delay of two days. The response of the
472 parametric model is smoothed compared to that determined by the non-parametric approach; the
473 small oscillations present on the latter for offsets longer than 16 days correspond to low-level noise
474 without physical meaning. This result demonstrates that karst phenomena are not always well
475 explained by physical laws, i.e., the parametric function. The recession of the normalized impulse
476 responses for the parametric and non-parametric functions (for lags >16 days) are very similar. This
477 result shows that the optimization procedure used in the Tempo software to calculate the
478 regularization parameter of the non-parametric function produces solutions that are smooth enough
479 to be compared to that obtained by a model of exponential decay. These results have reinforced
480 our belief that the hydrogeological functioning of the Lez karst system would be better characterized
481 by the use of a non-parametric impulse response.

482 As outlined by Long and Malher (2013), the modeling performance can be verified by using the
483 length of the modeling's validation period. A lengthy validation period with a good fit allows us to
484 evaluate the model's ability to predict the evolution of the system under other constraints (for
485 example climate change and/or resource management scenarios). The modeling approach used in
486 this study provides a very satisfactory reproduction of the daily time series of the groundwater level
487 measured since 1974, with a Nash criterion of 0.68 and a low standard deviation (0.06). The time
488 series calculated by the model are considered generally satisfactory, especially during periods of
489 aquifer drawdown when the pumped discharge is greater than the natural discharge in the absence
490 of pumping. In retrospect, these results validate the approach of reconstructing the natural flows of
491 the Lez. Nevertheless, for some hydrological cycles, the model appears sensitive to summer rainfall
492 episodes, when the calculated groundwater level may appear higher than the measured values. For

493 the period prior to 1983, the model tends to overestimate the drawdowns, which indicates that the
494 natural discharges estimated by the modeling are probably underestimated. Indeed, we previously
495 showed that the groundwater-level time series in the karst conduit are controlled solely by the
496 impulse function of the mobilized storage flow (Figure 8b), as soon as the groundwater level falls
497 below the H_0 reference level of 59.9 m a.s.l. The under- and over-estimations of the simulated low-
498 water groundwater levels ($H < 65$ m.a.s.l.) are therefore related to the mobilized storage flow, and
499 consequently to the natural flow estimated by modeling. An overestimation of the natural flows from
500 the Lez spring results in an underestimation of the mobilized storage flow. The transfer model
501 reflects this underestimation of Q_s by simulating groundwater levels higher than the measured
502 values (as with 1992, for example; Figure 9e). Conversely, an underestimation of the natural flow
503 leads to a simulation of groundwater levels below the measured values (as with 2006, for example;
504 Figure 9e).

505 In order to assess the model's sensitivity to the estimated natural discharge, the Lez groundwater
506 level was simulated assuming that the spring discharge is known to within $\pm 10\%$. The results of new
507 groundwater-level simulations show that the model is generally not very sensitive to the uncertainty
508 in the estimated Lez flow rates (figure not given). On the other hand, the results indicate that the
509 modeling bias, highlighted in some hydrological cycles, is caused by poor estimation of the effective
510 rainfall. The transfer model of Module 1 appears optimistic when significant rainfall events ($H > 25$
511 mm) occur during the summer low-water period. The simulated groundwater level during these
512 periods therefore appears overestimated because the simulated natural flows are overestimated.

513

514 **Area of pumping influence**

515 The area of the karst catchment in which recharge occurs is estimated to be 116 km², which
516 corresponds to the area of outcropping Jurassic limestone in the hydrogeological basin (100 km²), to

517 the contribution of occasional losses from temporary streams, and also to leakage from the overlying
518 Cretaceous. This area is comparable to the 120 km² determined by Thiery and Bérard (1983) and the
519 130 km² area parameterized by Fleury et al. (2008) in their reservoir model. The natural discharge of
520 the Lez spring estimated by modeling is 1870 l/s ($2\sigma = 590$ l/s, interannual average for the 1974–2010
521 period). Also for this period, the average interannual discharge of the residual overflow rate (Q_r) is
522 1030 l/s, the pumping rate (Q_p) is 995 l/s, and the mobilized storage flow (Q_s) is 150 l/s.

523 The hydrogeological model developed in this study provides interesting information on the structure
524 and flow regime of the Lez karst system. When the spring dries up ($H < 65$ m.a.s.l.), the time series of
525 the water level in a karst conduit is controlled by the mobilized storage flow, which is a function of
526 the pumping rate and the hydrogeological state of the karst in the absence of pumping (as described
527 by the natural flow of Lez estimated by modeling). The amount of drawdown as a function of the
528 volume pumped from the karst storage (V_s) is shown in Figure 11. It appears linearly related to the
529 volume of water pumped from the karst storage, the slope of the line being estimated at 112,800 m³
530 per meter of drawdown. In the pumped section of the water column (between 35 and 65 m.a.s.l.).
531 The results do not reveal any change in the hydrodynamic parameters with depth. For the water
532 column below 35 m.a.s.l., a modification of the hydraulic parameters cannot be excluded. Only a
533 pumping test in this section of the water column would provide the answer. The equivalent area of
534 karst storage mobilized by pumping has been estimated from the slope (112,800 m³/m \pm 1800 m³/m)
535 and the effective porosity (0.91%) calculated by Module 3, which characterizes the karst matrix
536 below the Cretaceous cover. Assuming an elliptical groundwater level surface within the matrix
537 tending toward the conduits, the area of pumping influence around the network of karst conduits is
538 estimated at 59 km², which is only a small fraction of the Lez spring's groundwater basin.

539 **Active management**

540 Hydrogeologically, reconstructing the Lez spring's natural flow makes it possible to characterize the
541 development of the aquifer's water resources since the onset of 'active management' based on

542 drilling (1983). Analysis of the reconstructed natural flows for the 1974–2010 period shows that the
543 volumes extracted annually are generally less than the annual recharge (Figure 11a). For the rainfall-
544 deficient years (1983, 1998, 2007), the pumped volumes are comparable to the volume of recharge
545 ($V_p \sim V_s$). The volumes of water pumped from the karst storage on average represent 18% (V_s/V_p
546 ratio) of the mean annual pumped volumes (32.3 million m^3 ; Figure 11b). The V_s/V_p ratio rises to
547 30% in the rainfall-deficient years. Figure 11b also clearly shows the moment at which the active
548 management of the Lez resource was modified (1983).

549 The annual trend of the different volumes (V_n , V_p , and V_s) and the V_s/V_p ratio are shown in Figure
550 12 (interannual average over the period 1983–2010). The volumes of water pumped from the karst
551 storage vary during the year (Figure 12b), with the V_s/V_p ratio being highest in August and
552 representing an interannual average of 55% of the pumped volume for this month. The interannual
553 average V_s/V_n ratio reaches 170% in August, indicating that the volume of water pumped from the
554 karst storage is much higher than the average volume of water that would have flowed naturally in
555 the absence of pumping (interannual average of +70%). The box plot (Figure 12d, e) illustrates the
556 range in variability of the calculated ratios for the summer months (July, August, and September). For
557 the rainfall-deficient years corresponding to the dry quinquennials (decile 0.8), the V_s/V_p and V_s/V_n
558 ratios are respectively 70% and 235% in August.

559 **Conclusion**

560 The methodological approach developed in this study is based on breaking down the head variations
561 of a karst system subjected to pumping, and on modeling the outlet discharges using transfer
562 models. This approach has made it possible to model the groundwater level fluctuations in different
563 compartments of the Lez karst aquifer, the residual flows at the spring, and the pumped karst
564 storage flow. It is an overall semi-distributed approach enabling the aggregation of the hydrological
565 responses of the various compartments interacting with each other. It constitutes a modeling

566 alternative when (i) the complexity of the karst aquifer necessitates a more elaborate approach than
567 a simple lumped model, and (ii) the data necessary for a distributed model are not available. It has
568 also helped to reconstruct the natural flow of the spring in the absence of pumping. The uncertainty
569 generated by this last aspect was the subject of a sensitivity analysis that showed the model to be
570 relatively insensitive to the uncertainty in calculating the naturalized flow.

571 The models developed in this study reveal that the aquifer is not being overexploited. They will
572 eventually allow the testing of various resource-management scenarios and assessments of the
573 impacts on a) the time series of the karst system's total head (karst conduit and matrix), and b) the
574 time series of residual flow. The data acquired will make it possible to evaluate the impacts of
575 climate change and anthropogenic pressures on the short- and long-term water changes in the water
576 resources, using wavelet analysis

577 **Acknowledgements**

578 This work was carried out under the Lez-GMU Project, in the framework of the BRGM's EAUR15
579 research program. The authors thank the Montpellier Agglomeration, the Water Agency Rhone-
580 Méditerranée-Corse, and the Hérault General Council, as well as the BRGM, for their funding
581 support.

582 The authors also wish to thank Barbara Malher and the anonymous referee for their helpful
583 comments, which led to substantial improvements in the manuscript.

584

585

586 **Figure captions**

587 Figure 1: Simplified map of the lithotectonic units in the study area. Jurassic limestone outcrops
588 correspond to the main recharge zones.

589 Figure 2: Interpretative geological cross section (NW – SE) of the Lez aquifer and architecture of the
590 models used to simulate (1) the natural Lez spring discharge from effective rainfall; (2) the water
591 level in the karst conduit (Suquet borehole) located close to the intermittent Lirou karst spring fed by
592 effective rainfall; (3) water level in the matrix (Laudou piezometer) fed by effective rainfall and the
593 storage flow (Qs) mobilized by pumping; and (4) water level in main conduit (Lez borehole) fed by
594 effective rainfall, Jurassic contribution from aquifer located west of the Matelles Fault (2), matrix
595 contribution (3), and contribution of the storage flow (Qs) mobilized by pumping.

596 Figure 3: Cross section of the karst network upstream of the Lez spring.

597 Figure 4: Changes in water levels in the karst conduit (Lez and Suquet boreholes, Claret piezometer)
598 and matrix (Laudou piezometer) for the period 2000–2009.

599 Figure 5: Module 1 - Inverse modelling of the natural discharge of the Lez spring on a daily time step.
600 The observed and modelled natural discharges for flood and high-water periods are plotted for both
601 the calibrated period (a) and simulated period (b). The Nash coefficient is 0.67 for the high waters of
602 the 1987–2007 period. The natural discharges for the low-water periods are calculated from the
603 impulse response of the slow component (e). The impulse responses of the infiltration threshold
604 $\Omega(t)$ to rainfall and air temperature are also plotted (c, d), as are the slow and fast normalized
605 impulse responses of the effective recharge (e, f).

606 Figure 6: Module 2 - Inverse modeling of the water level in the karst conduit located close to the
607 intermittent Lirou spring (Suquet borehole) on a daily time step. (a) Infiltration threshold used in
608 calculating the effective rainfall. (b) Comparison between the observed and modeled water levels
609 from non-parametric model. (c) Slow normalized impulse response to effective rainfall (the fast
610 impulse response not detected) for non-parametric and parametric models. (d) Water level
611 simulated for the 2000–2012 period.

612 Figure 7: Module 3 - Inverse modeling of the matrix water level (Laudou piezometer) on a daily time
613 step. (a) Comparison between the observed and modeled water levels. (b) Slow normalized impulse
614 response to effective rainfall (fast impulse response not detected) and impulse response to the
615 mobilized karst storage flow (Q_s). (c) Hydrograph separation of the two water-level related
616 components (Fig. 2) – piezometric variations are expressed relative to the reference level ($H_0 = 80$
617 m.a.s.l.). (d) Water levels simulated for the 2000–2011 period.

618 Figure 8: Module 4 - Inverse modeling of the water level measured in the main karst conduit at Lez
619 (Lez borehole) on a daily time step. (a) Comparison between the observed and modeled water levels
620 for the calibrated period. (b) Hydrograph separation of the four water-level related components of
621 the transfer model (Fig. 2) – piezometric variations are expressed relative to the reference level ($H_0 =$
622 59.9 m.a.s.l.). (c) Normalized impulse responses of the four components.

623 Figure 9: (a) Comparison between the observed and modeled water levels for the 1974–2010 period.
624 (b) Effective rainfall calculated for the 1974–2010 period. (c) Natural spring discharge (Q_n), pumping
625 discharge (Q_p), and pumped karst storage flow (Q_s) for the 1974–2010 period. (d) Residual spring
626 discharge (Q_r). (e) Detail of the transfer model inputs of the water levels measured in the main karst
627 conduit (water levels modeled by the transfer models 2 and 3 (Fig. 2))

628 Figure 10: Cross correlogram of piezometric data used to describe the hydrodynamics of the Lez karst
629 system.

630 Figure 11: Evolution of the average monthly drawdown in terms of the pumped karst storage
631 volume.

632 Figure 12: Evolution of the various annual volumes of water for the 1974–2010 period. The evolution
633 of the V_s/V_p ratio is plotted at annual scale.

634 Figure 13: (a) Trends of the various volumes of water (average interannual monthly for the 1983–
635 2010 period). The trends of the V_s/V_p and V_s/V_n ratios calculated for the 1983–2010 period are
636 plotted at monthly scale (b, c). The box plots of the V_s/V_p and V_s/V_n ratios are given for the summer
637 months (d and e).

638

639

640 **References**

641 Aquilina L., Ladouche B., Doerfliger N., 2005. Recharge processes in karstic systems investigated through the
642 correlation of chemical and isotopic composition of rain and spring waters. *Appl.Geochem.* 20(12):2189-206.

643 Aquilina L., Ladouche B., Doerfliger N., 2006. Water storage and transfer in the epikarst of karstic systems
644 during high flow periods. *Journal of Hydrology* 327(3-4):472-85.

645 Avias J., 1992. Contrôles Géologiques des systèmes aquifères karstiques (s.a.k) de type méditerranéen :
646 l'exemple du S.A.K de la source du Lez-France-(Karstic aquifers of Mediterranean type, geological controls : Lez
647 Spring (North-Montpellierant karsts, France) example. *Hydrogeology of Selected Karst Regions, AIH, Volume*
648 13:89-113.

649 Avias J., 1995 - Gestion active de l'exurgence karstique de la source du Lez (Hérault, France) 1957-1994.
650 *Hydrogéologie* 1:113-127.

651 Bakalowicz M., 2005. Karst groundwater; a challenge for new resources. *Hydrogeol. J.* 13(1):148-60.

652 Bérard P., 1983. Alimentation en eau de la Ville de Montpellier. Captage de la source du Lez, commune de St-
653 Clément (Hérault). Etude des relations entre la source et son réservoir aquifère. Rapport n°2, Définition des
654 unités hydrogéologiques. BRGM Report 83 SGN 325 LRO, 27 p.

655 Birk S, Liedl R, Sauter M (2000) Characterization of gypsum aquifers using a coupled continuum-pipe flow
656 model. In: Stauffer F, Kinzelbach W, Kovar K, Hoehn E (eds) Calibration and reliability in groundwater modelling,
657 vol 265. IAHS Publication, Idyllwild, pp 16–21

658 Birk S., Liedl R., Sauter M., Teutsch G., 2003. Hydraulic boundary conditions as a controlling factor in karst
659 genesis: A numerical modeling study on artesian conduit development in gypsum. *Water Resources Research*
660 39, Art. No. 1004.

661 Clauzon G., Suc J.P., Popescu S.M., Marunteanu M., Rubino J.L., Marinescu F., Melinte M.C., 2005. Influence of
662 Mediterranean sea-level changes on the Dacic Basin (Eastern Paratethys) during the late Neogene: the
663 Mediterranean Lago facies deciphered. *Basin Research* 17:487-462.

664 Conroux Y., 2007. Caractérisation du fonctionnement hydrodynamique de l'aquifère karstique du Lez à l'état
665 naturel. Mémoire Master 2, Univ. Avignon 2, 151 p.

666 Cornaton F. and Perrochet P., 2002. Analytical 1D dual-porosity equivalent solution to 3D discrete single-
667 continuum models. Application to karstic spring hydrograph modelling. *Journal of Hydrology*, 262, 165-176

668 Denic-Jukic V., Jukic D., 2003. Composite transfer functions for karst aquifers. *Journal of Hydrology* 274:80-94.

669 Dietrich C.R., Chapman T.G., 1993. Unit graph estimation and stabilization using quadratic programming and
670 difference norms. *Water Resour Res.* 29(8):2629-2635.

671 Dorfliger N., Fleury P., Ladouche B., 2009. Inverse Modeling Approach to Allogenic Karst System
672 Characterization. *Ground Water* 47(3):414-26.

673 Doherty J. and Skahill B.E., 2006. An advanced regularization methodology for use in watershed model
674 calibration. *Journal of Hydrology* 327, 564– 577, doi:10.1016/j.jhydrol.2005.11.058

675 Dreiss S.J., 1983. Linear unit-response functions as indicators of recharge areas for large karst springs. *J. Hydrol.*
676 61:31–44.

677 Drogue C., Grillot J.C., 1976. Structure géologique et premières observations piézométriques à la limite du sous-
678 système karstique de Terrieu (Périmètre expérimental). *Ann. Sc. Univ. Besançon* 25:195-210.

679 Fleury P., Ladouche B., Conroux Y., Jourde H., Dörfliger N., 2008. Modelling the hydrologic functions of a karst
680 aquifer under active water management – the Lez spring. *Journal of Hydrology* 365:235–243.
681 doi :10.1016/j.jhydrol.2008.11.037

682 Goldscheider N. and Drew D, 2007. *Methods in Karst Hydrogeology.* Taylor & Francis. International
683 Contributions to Hydrogeology

684 Guilbot A., 1975. Modélisation des écoulements d'un aquifer karstique (liaison pluie debit). Applications aux
685 bassins de Saugras et du Lez. Thesis, Université Montpellier II Sciences et Techniques du Languedoc, 110 p.

686 Jukić D., Denić-Jukić V., 2009. Groundwater balance estimation in karst by using a conceptual rainfall–runoff
687 model. *Journal of Hydrology* 373(3-4):302-315.

688 Karam Y., 1989. Essai de modélisation des écoulements dans un aquifère karstique. Exemple de la source du
689 Lez (Hérault, France). Thesis, Université Montpellier II Sciences et Techniques du Languedoc, 210 p.

690 Lacas J.L., 1976. Introduction à la méthodologie d'étude et d'utilisation des champs hydrothermiques des
691 aquifères karstiques: d'après l'exemple du site de l'exurgence de la source du Lez (Hérault, France). Université
692 Montpellier II Sciences et Techniques du Languedoc, CERGA. 68 p.

693 Liedl R., Sauter M., Hückinghaus D., Clemens T., Teutsch G., 2003. Simulation of the development of karst
694 aquifers using a coupled continuum pipe flow model. *Water Resources Research* 39, Art. No. 1057.

695 Long A. J. and Mahler B. J. (2013). Prediction, time variance, and classification of hydraulic response to recharge
696 in two karst aquifers. *Hydrol. Earth Syst. Sci.*, 17, 281–294, doi:10.5194/hess-17-281-2013

697 Mangin A., 1975 - Contribution à l'étude hydrodynamique des aquifères karstiques (Contribution to
698 hydrodynamic study of karstic aquifers) Thèse de Docteur en Sciences, Université de Dijon ;Annales de
699 Spéléologie, 1974,29,3, pp283-332 ;1974,29,4, pp495-601 ;1975 , 30, 1, pp21-124.

700 Marechal J.C., Ladouche B., Doerfliger N., Lachassagne P., 2008. Interpretation of pumping tests in a mixed flow
701 karst system, *Water Resources Research* 44(5): doi: 10.1029/2007WR006288

702 Martínez-Santos P., Andreu J.M., 2010. Lumped and distributed approaches to model natural recharge in
703 semiarid karst aquifers. *Journal of Hydrology* 388(3–4):389-98. doi:10.1016/j.jhydrol.2010.05.018

704 Mazzili N. (2011). Sensibilité et incertitude de modélisation sur les bassins versants a forte composante
705 karstique, PhD thesis, Université Montpellier II, Sciences et Techniques du Languedoc, 210p

706 Mazzilli N., Guinot V., & Jourde H. Sensitivity analysis of conceptual model calibration to initialisation bias
707 (2011). Application to karst spring discharge models. *Advances in Water Resources* 42(8), 1-16.
708 doi:10.1016/j.advwatres.2012.03.020.

709 Moussu F., Oudin L., Plagnes V, Mangin A.and Bendjoudi H., 2011. A multi-objective calibration framework for
710 rainfall–discharge models applied to karst systems *Journal of Hydrology* 400 364–376.
711 doi:10.1016/j.jhydrol.2011.01.047

712 Nash J.E., Sutcliffe J.V., 1970. River flow forecasting through conceptual models part I -A discussion of
713 principles. *Journal of Hydrology* 10(3):282-290.

714 Padilla A. and Antonio Pulido-Bosch A., 2008.Simple procedure to simulate karstic aquifers. *Hydrol. Process.* 22,
715 1876–1884. doi: 10.1002/hyp.6772

716 Paloc H., 1979. Alimentation en eau de la ville de Montpellier. Captage de la source du Lez, commune de St.
717 Clément (Hérault). Etude documentaire préalable à l'établissement des périmètres de protection. BRGM Note
718 de synthèse 79SGN319LRO, 47p.

719 Paloc H., 1987. Première évaluation des capacités de production d'eau par pompage du siphon amont de la
720 grotte dite "Du Grand Bouldou". Commune des Matelles (Hérault). BRGM Report 87SGN604LRO, 44p.

721 Pinault J.L., 2001. Manuel utilisateur de TEMPO, logiciel de traitement et de modélisation des séries
722 temporelles en hydrogéologie et en hydrogéochimie. BRGM Report RP-51459-FR, 221 p.

723 Pinault J.L., Pauwels H., Cann C., 2001a. Inverse modeling of the hydrological and the hydrochemical behavior
724 of hydrosystems; application to nitrate transport and denitrification. *Water Resour.Res.* 37(8):2179-90.

725 Pinault J.L., Plagnes V., Aquilina L., Bakalowicz M., 2001b. Inverse modeling of the hydrological and the
726 hydrochemical behavior of hydrosystems; characterization of karst system functioning. *Water Resour. Res.*
727 37(8):2191-204.

728 Pinault J.L, Doerfliger N., Ladouche B., Bakalowicz M., 2004. Characterizing a coastal karst aquifer using an
729 inverse modeling approach; the saline springs of Thau, southern France. *Water Resour. Res.* 40(8):17.

730 Pinault J.L, Amraoui N., Golaz C., 2005. Groundwater-induced flooding in macropore-dominated hydrological
731 system in the context of climate changes. *Water Resour. Res.* 41(5): W05001, doi:10.1029/2004WR003169.

732 Pinault J.L, Schomburgk S, 2006. Inverse modeling for characterizing surface water/groundwater exchanges.
733 *Water Resour.Res.* 42(8): W08414, doi:10.1029/2005WR004587.

734 Pinault J.L, Allier D, 2007. Regionalization of rainfall for broad-scale modeling; an inverse approach. *Water*
735 *Resour.Res.* 43(9): W09422, doi:10.1029/2006WR005642,.

736 Sivapalan M., 2003. Prediction in ungauged basins: a grand challenge for theoretical hydrology. *Hydrol. Process.*
737 17:3163–3170. doi: 10.1002/hyp.5155

738 Teutsch G., Sauter M., 1998. Distributed parameter modeling approaches in karst-hydrological investigations.
739 *Bull. Hydrogéol.* 16:99-109.

740 Thierry D., Berard P. & Camus A., 1983. Captage de la source du Lez : étude des relations entre la source et son
741 réservoir aquifère. *Ed. BRGM, 83 SGN 167 LRO, 95 pp.*

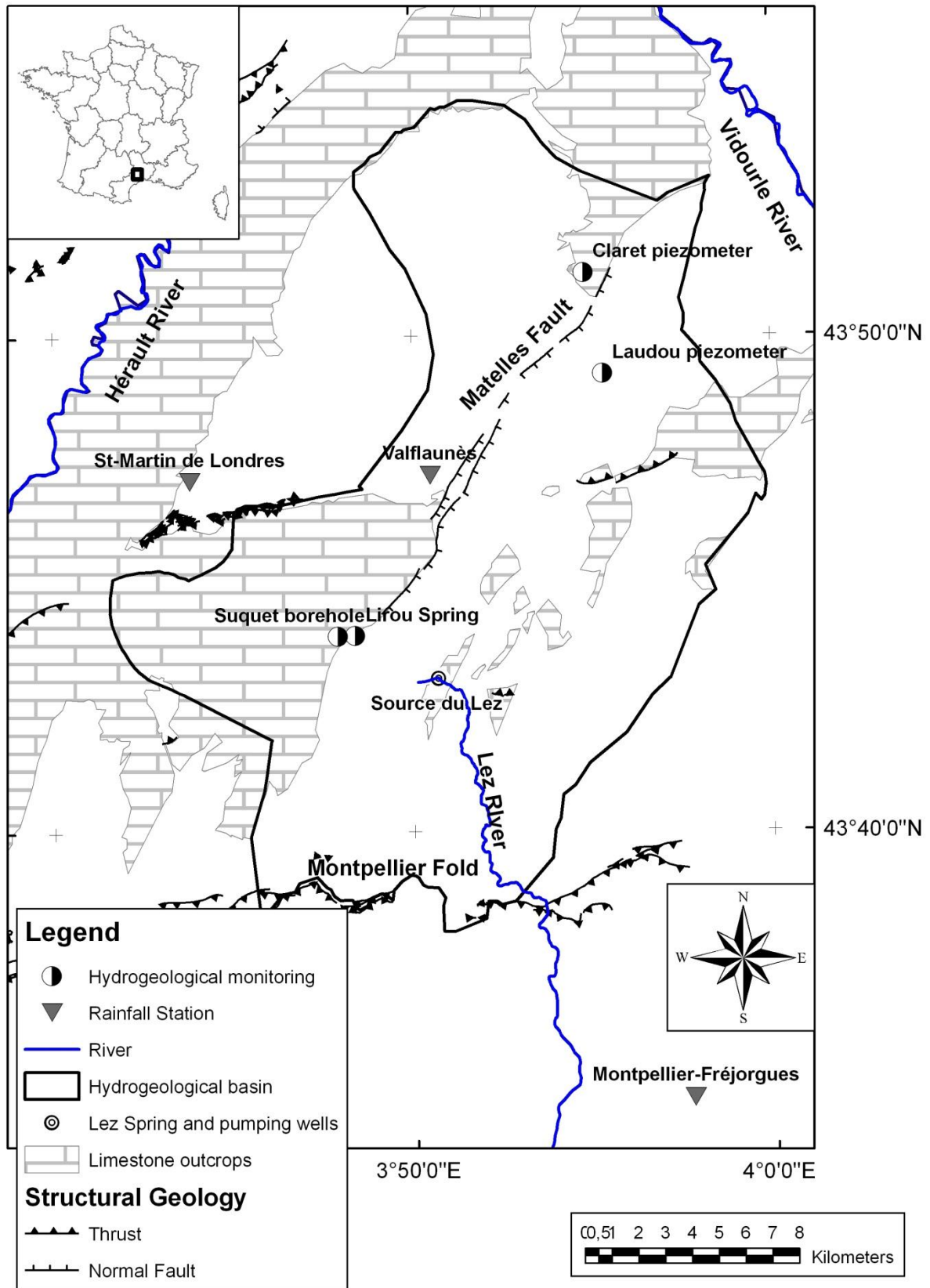
742 Tikhonov A., Arsenine V., 1976. Méthodes de résolution de problèmes mal posés. MIR, Moscow.

743 Tikhonov A.N., Goncharsky A.V. (Eds.), 1987. Ill-Posed Problems in the Natural Sciences. MIR, Moscow.

744 Von Asmuth, J. R., Bierkens M. F. P. and Maas K., 2002. Transfer function-noise modeling in continuous time
745 using predefined impulse response functions, *Water Resour. Res.*, 38(12), 1287, doi: 10.1029/2001WR001136

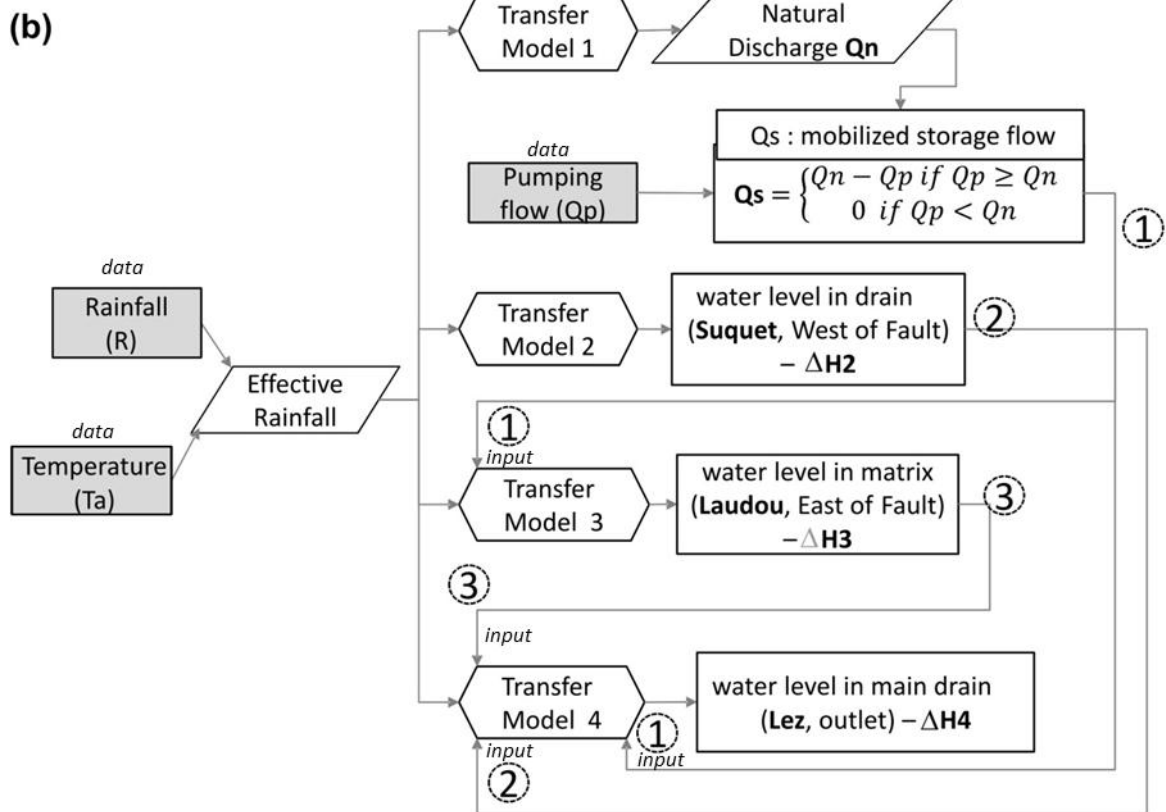
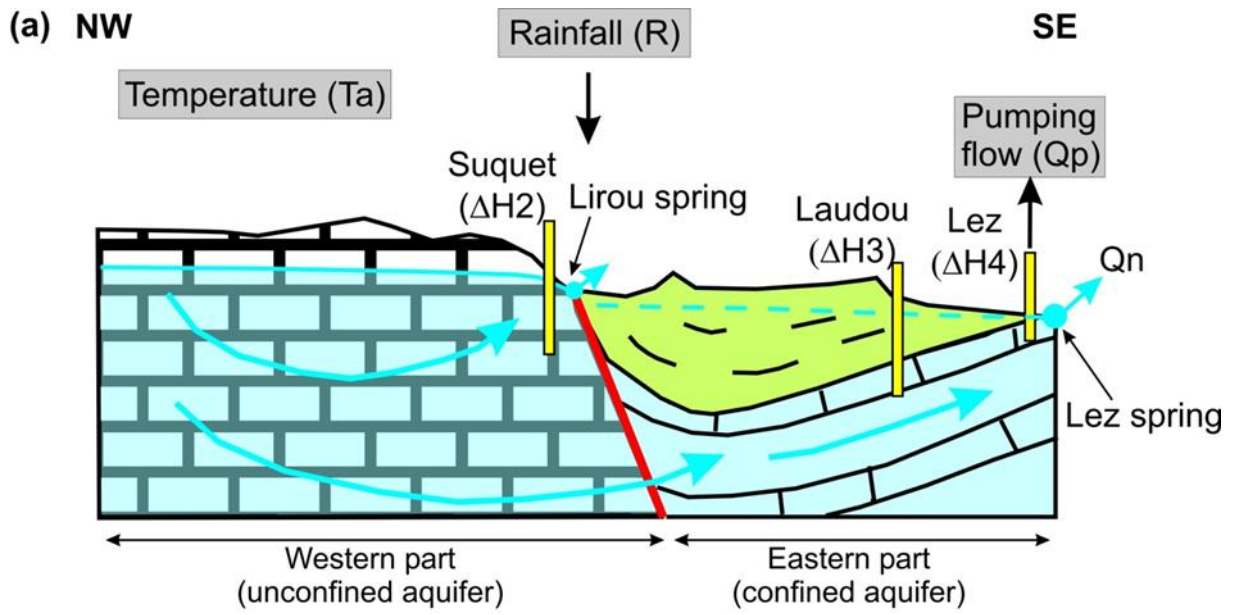
746

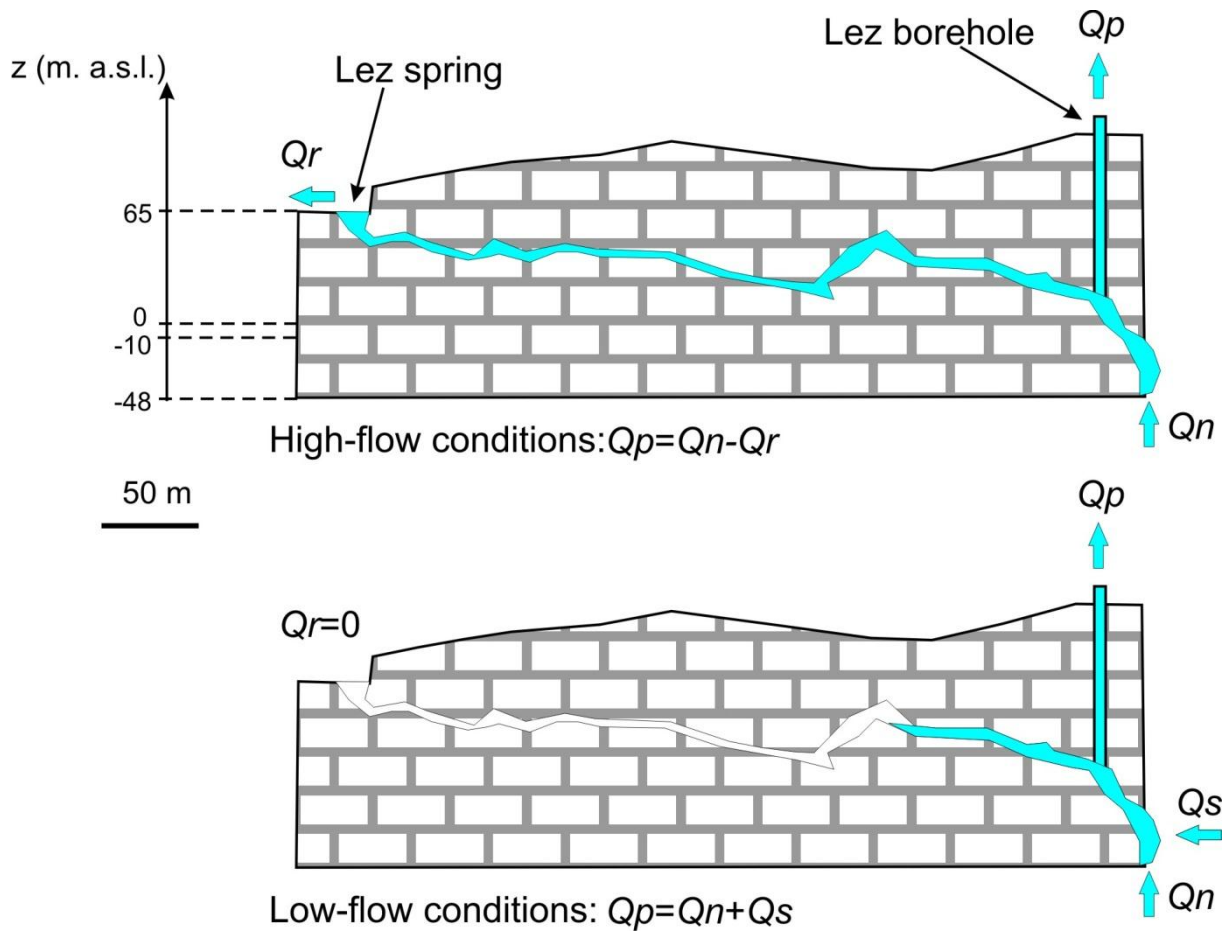
747



748

749 Figure 1.





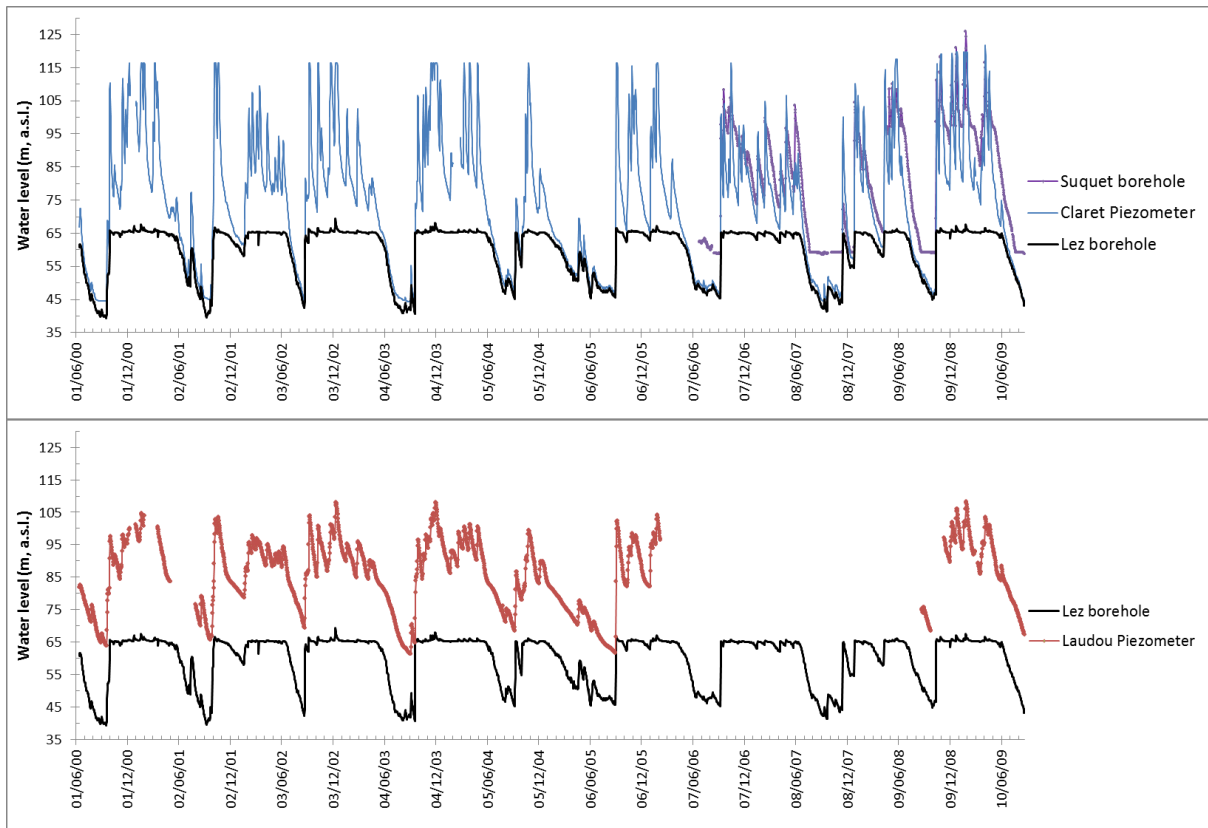
753

754

755 Figure 3

756

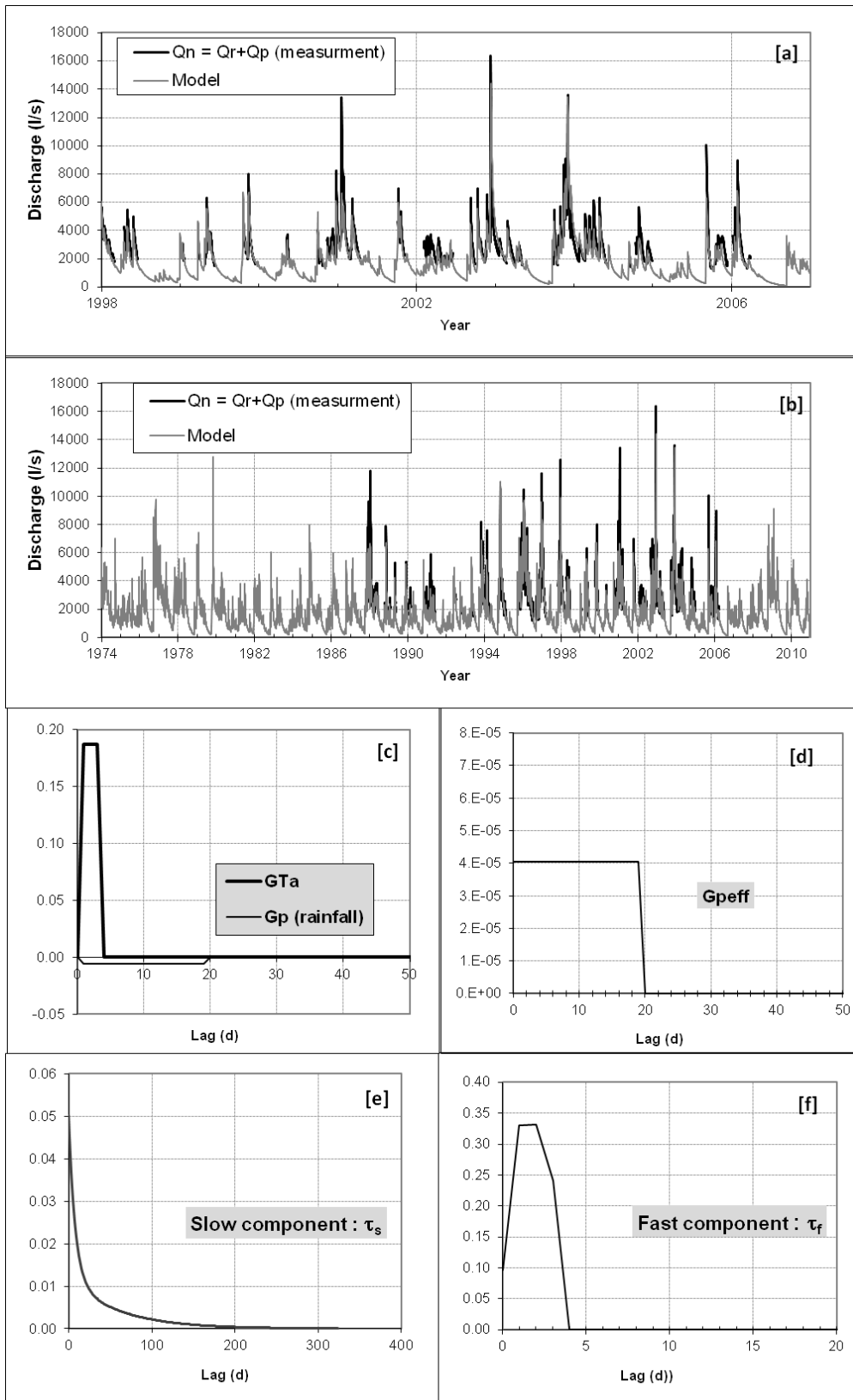
757



758

759 Figure 4

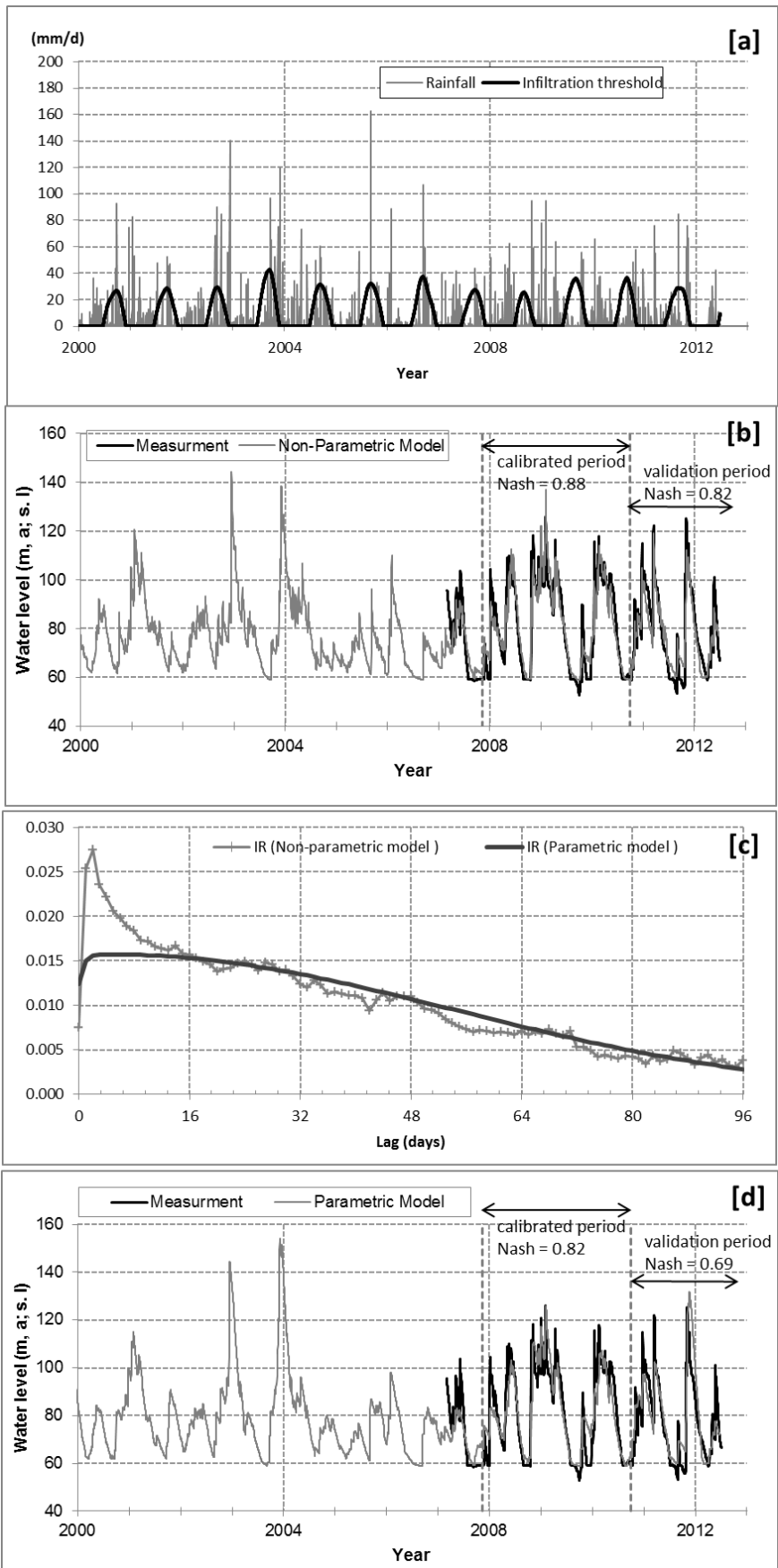
760



761

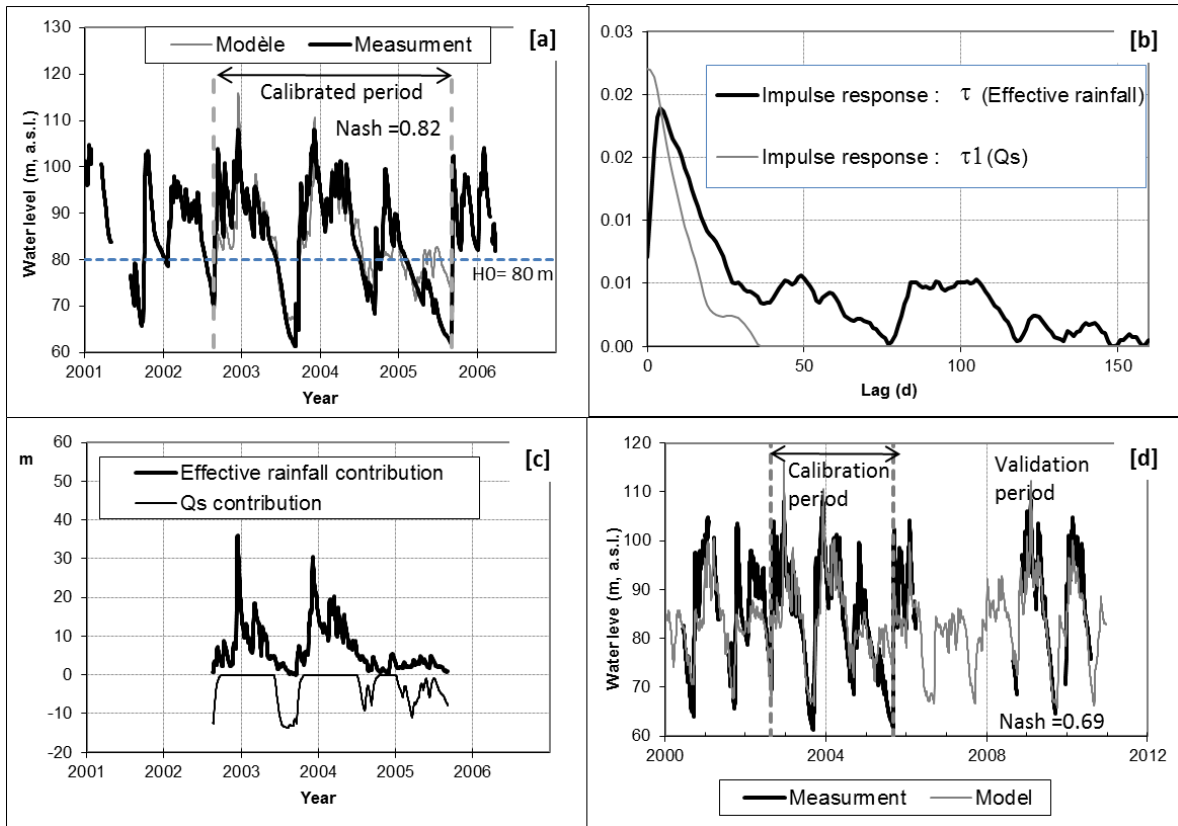
762 Figure 5

763



764

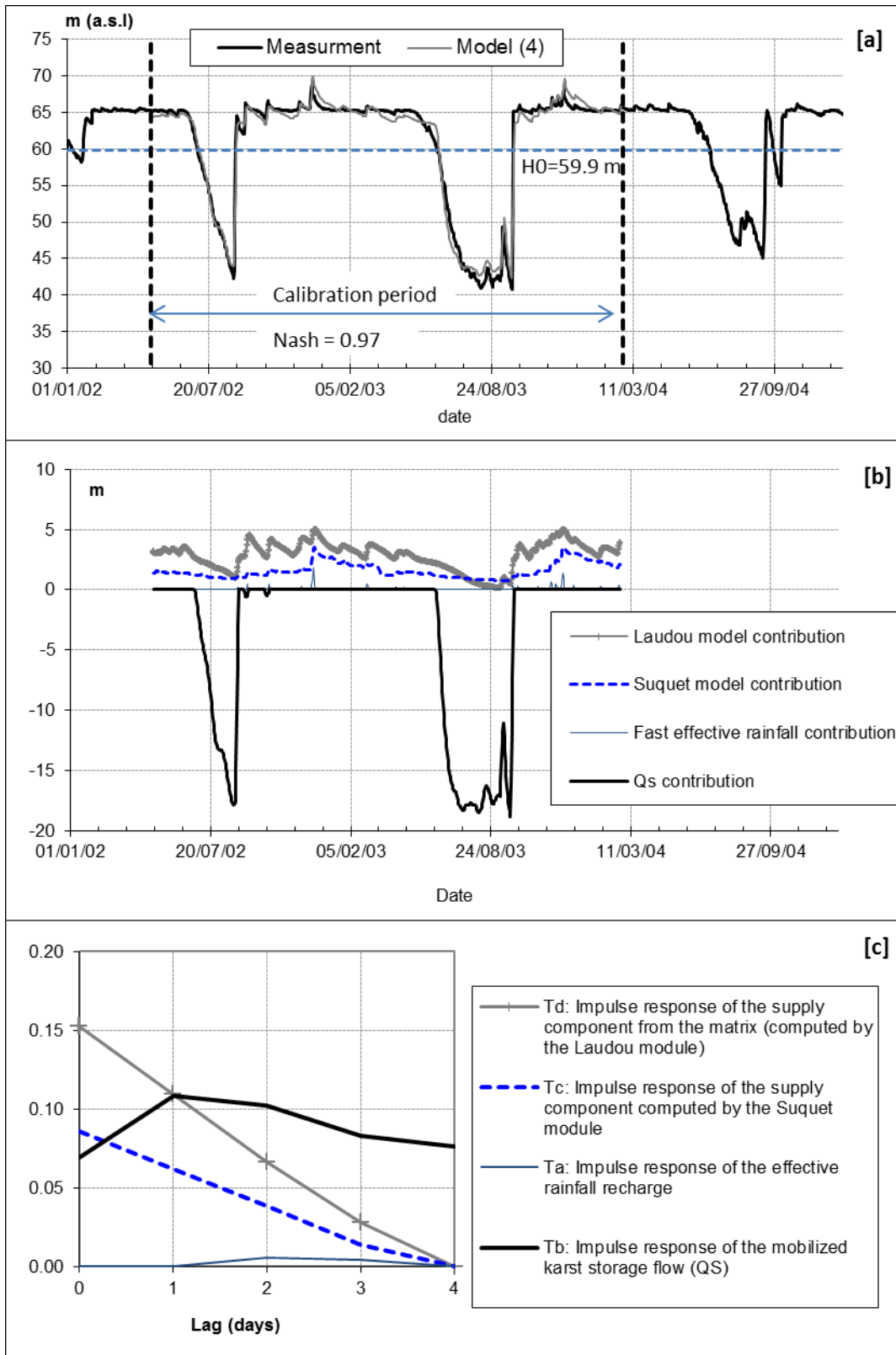
765 Figure 6



766

767 Figure 7

768



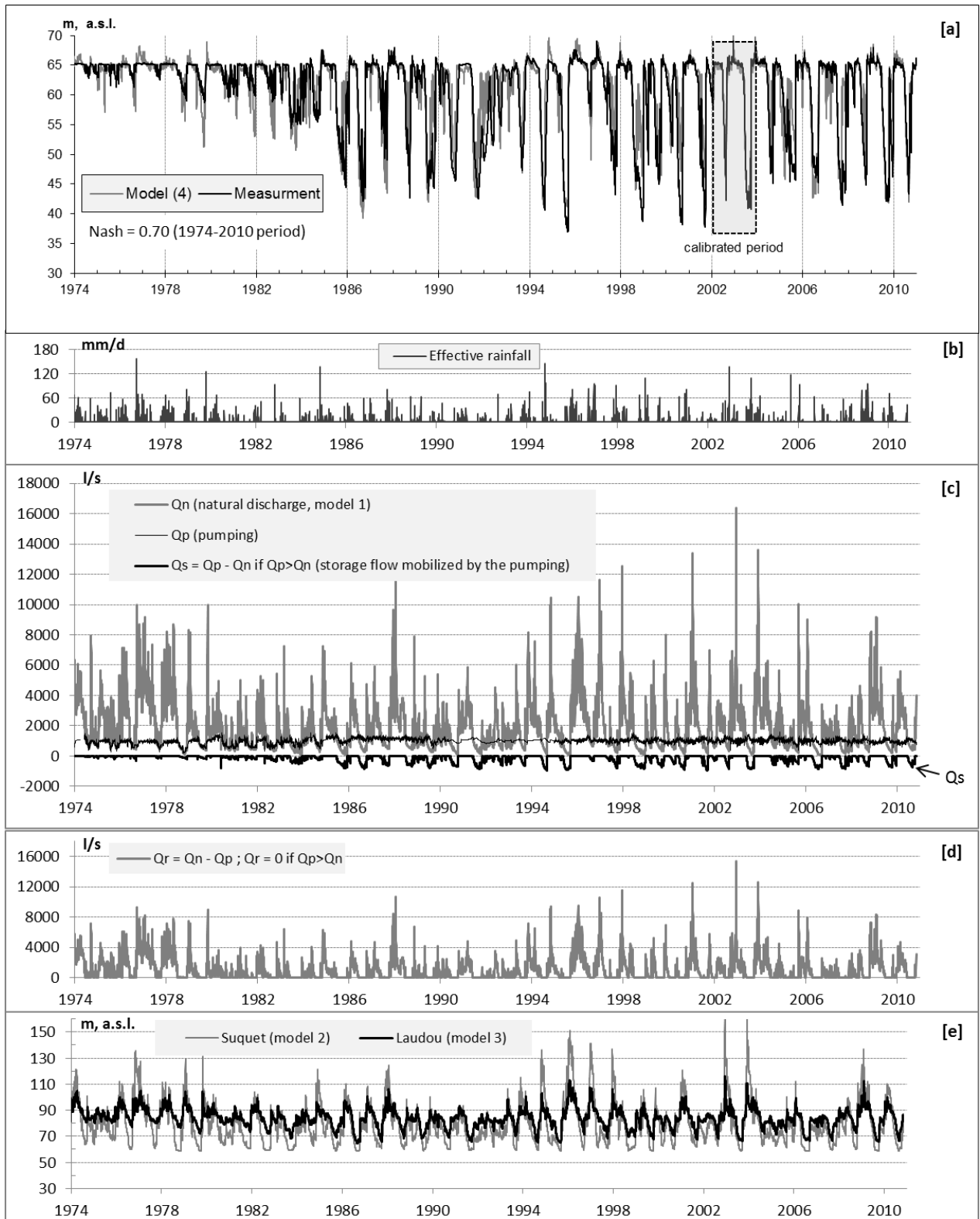
769

770 Figure 8

771

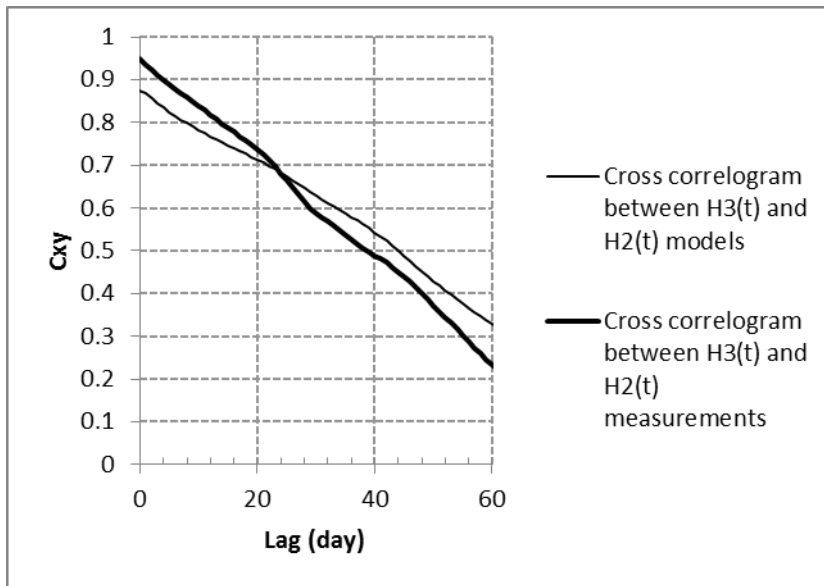
772

773



774

775 Figure 9.

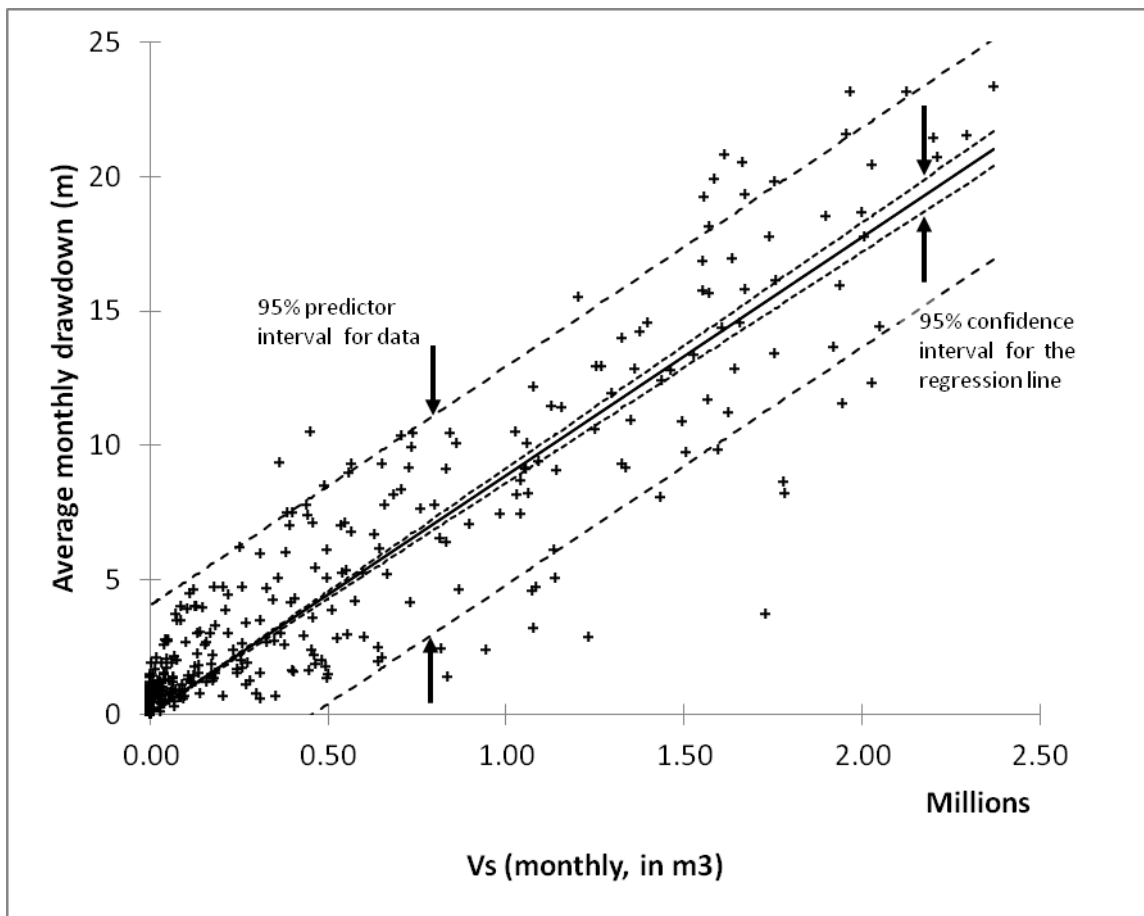


777

778 Figure 10

779

780

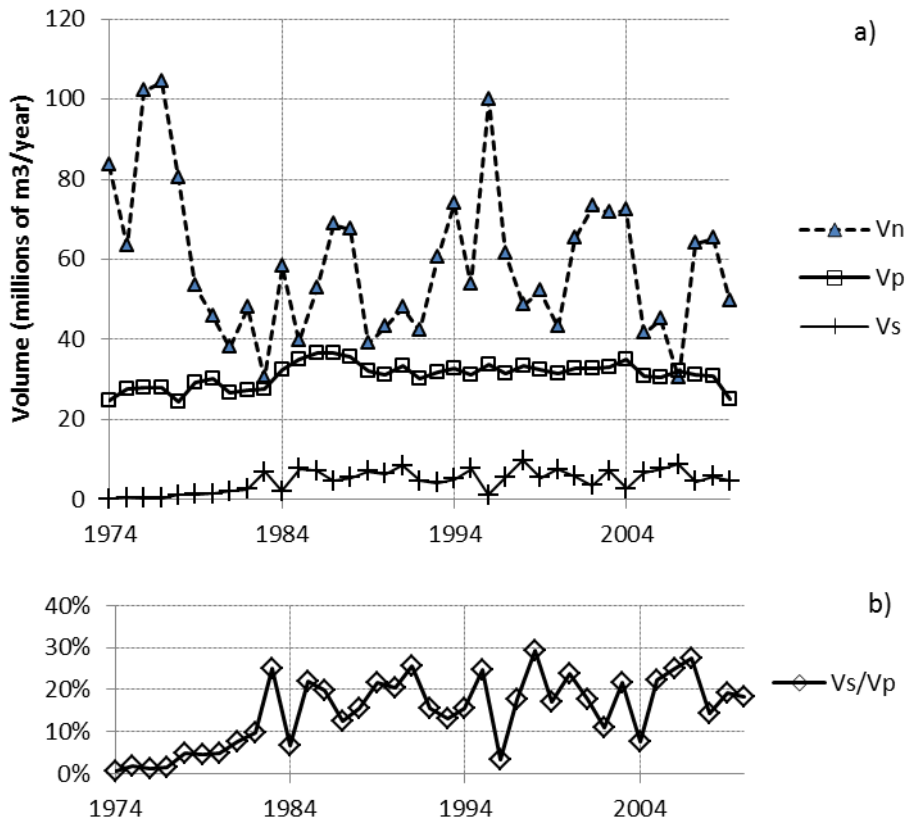


781

782 Figure 11

783

784

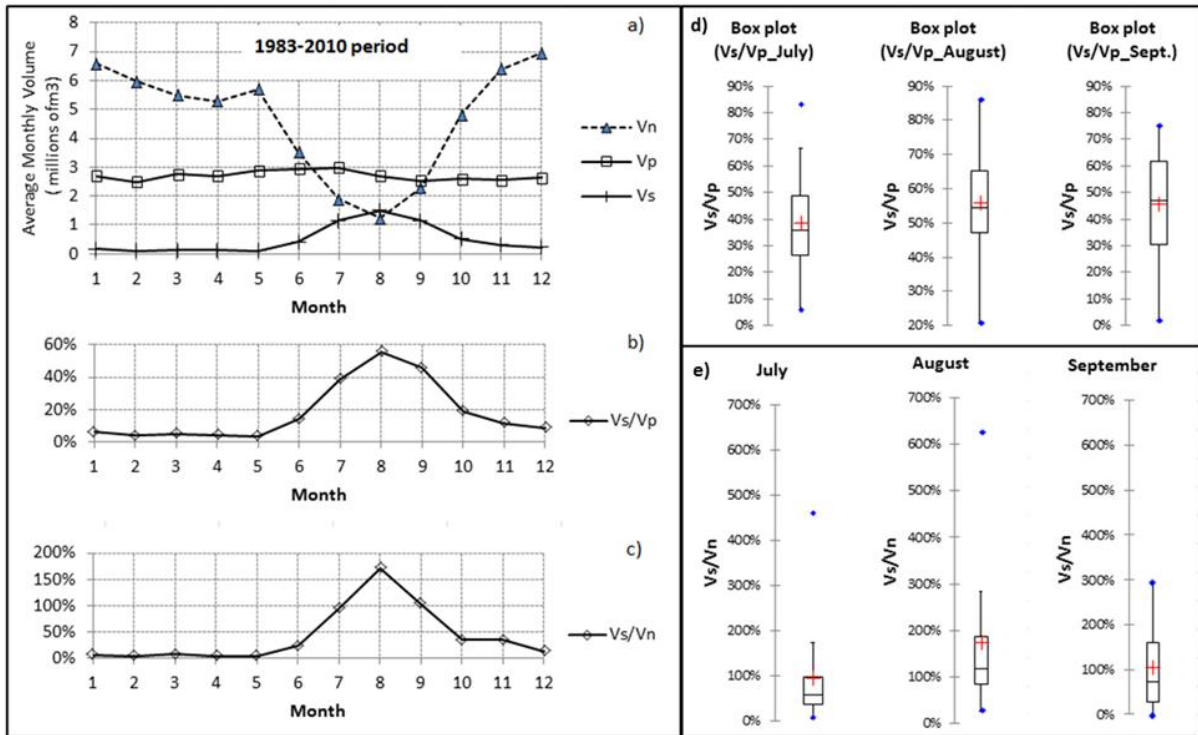


785

786 Figure 12

787

788



789

790 Figure 13.

791

792