A numerical approach for compiling full Physical Supply-Use Tables (PSUTs) under conflicting information

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ABSTRACT

Physical Supply-Use Tables (PSUTs) provide a comprehensive accounting of anthropogenic material flows within the economy and in interaction with the natural environment. Balanced PSUTs most often subsequently need to be converted to Physical Input-Output Tables (PIOTs) in order to address environmental issues. PSUT compilation, including data mining and mass balancing, requires large scale efforts. At the same time the benefits gained from PIOTs (in terms of environmental information and modelling) do not seem to be convincing enough for the National Statistical Institutes to plan their large scale production [8]. Accordingly there is a strong need to limit the cost and time required for PSUTs and PIOTs compilation while at the same improving their reliability.

This work proposes a numerical approach to balance full PSUTs under conflicting information using the notation of constrained optimization. The mass balancing identities of PSUTs, in terms of products and activities, are applied in a mathematical technique which fulfills all requirements of constrained optimization techniques. Following the theoretical framework as defined in a first part, a tentative PSUT for the Netherlands 2006 is presented as a case study (being validated). Such an approach could constitute a major advance for the practice of compiling PSUTs before deriving PIOTs, since it aims to remove the necessity of manually tracing conflicting information. In addition, this article gives some guidelines for future research (e.g. a full PSUT compiled for France for the year 2006) which could help to make PIOTs more relevant and cost-effective.

Keywords: Constrained optimization technique, Mass balance, Physical Input-Output Tables (PIOTs), Physical Supply-Use Tables (PSUTs)

INTRODUCTION

Physical Supply-Use and Input-Output Tables (PSUTs and PIOTs) provide a comprehensive picture of anthropogenic material flows within the economy and in interaction with the natural environment. They complement the corresponding monetary tables (Monetary Supply-Use Tables, MSUTs) by registering flows of physical products, extraction of materials from nature, supply and use of wastes, emissions to nature and stock changes [3, 5, 9].

There are three main advantages that can be identified by using PSUTs and PIOTs: integration of physical data, improvement of monetary statistics and environmental-economic modelling [8]. Of course these advantages come at a price. The data requirements and cost of producing PSUTs and PIOTs are large, and increase when the tables become more complex. We propose a mathematical model based on which the PSUTs and PIOTs can be produced more cost-efficiently. Such a technique is currently being developed by the writers of the present paper. It includes a cost-saving strategy consisting of data reconciliation & error estimation.
Data reconciliation is a technique to improve the quality of measured data. These measurements are inherently inaccurate or subject to failures. Using erroneous data for accounting analysis and decision-making may yield distorted conclusions and result in improper decisions [15]. Accurate data is therefore essential for compiling PSUTs and subsequently analyzing material flows. One measure of data inaccuracy is the consistency with regard to the mathematical models describing the accounting system. Among the more classical models used for describing a functioning are the balance relationships (mass, component, species, enthalpy, etc.). If all of these models are structurally perfectly known, some of them depend on parameters which are difficult to assess. Therefore, it becomes very hazardous and mathematically not correct to reconcile operation data with regards to an uncertain model without taking this last fact into account. We propose to take this information into account in the reconciliation procedure, assuming some knowledge about the precision of the values of the input parameters.

This paper provides an overview of the literature on PSUTs and PIOTs. It also proposes an optimization model (currently under validation) which could help to make the PSUTs and PIOTs more relevant and cost-effective. Rudimentary PSUTs for the Netherlands for the year 2006 are only presented as an illustration for future applications. In a more prospective view, this paper proposes another approach for evaluating uncertainty related to model input parameters: the possibilistic or fuzzy approach [6].

SUPPLY-USE AND INPUT-OUTPUT TABLES

General framework
The framework of PSUTs and PIOTs, including their accounting identities, has been widely detailed in the literature, in particular by [3, 8, 9, 17]. We consider the latter’s description and notations in what follows. Every table mentioned below corresponds to a specified period and geographical area (e.g. France for the year 2006).

The Supply matrix, \( V \), of dimensions activities by products, reports the supply of products per human activity. The Use matrix, \( U \), of dimensions products by activities, details the intermediary consumptions of products per human activity. These two tables are completed by the Import and Export vectors, \( N \) and \( E \), of dimensions products by one, which report the exchanges of products with the rest of the world. Finally the vector \( Y \), of dimensions products by one, stands for the final consumption vector. This first set of tables is traditionally accounted for in monetary terms (i.e. MSUTs) and is correspondingly reported in physical units in PSUTs.

PSUTs additionally include the environment as a source of raw materials (matrix \( R \) of dimensions resources by activities) and as a sink for residuals and emissions (respectively matrices \( W_v \) and \( B \) of dimensions products by activities and emissions by activities). Finally, \( W_u \) and \( \Delta S \), of dimensions products by activities, respectively represent the use of residuals and the addition to stocks of products and residuals.

The accounting identities that structure the PSUTs are based on the material balance principle (Figure 1). On the one hand, on a product perspective:

\[
V + N = U + Y + E \tag{1}
\]

And on the other hand, on an activity perspective:

\[
V + W_v + B + \Delta S = U + W_u + R \tag{2}
\]

The balanced PSUTs may finally be converted into PIOTs by using one of the following assumptions: the product technology assumption, the industry technology assumption, and the assumption of fixed industry sales structure or the assumption of fixed product sales structure. This step is widely detailed and discussed in the literature [2] and will not be considered further in this paper. The derived PIOTs report in columns the intermediary consumptions of products,
emissions, resource consumptions, stocks changes, waste generation and use associated with the production of one additional unit (e.g. 1 ton) of the corresponding product or activity.

<table>
<thead>
<tr>
<th>Balanced PSUT</th>
<th>Activities</th>
<th>Import</th>
<th>Needs Fulfilment</th>
<th>Export</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Products</td>
<td>V'</td>
<td>N</td>
<td></td>
<td></td>
<td>Q</td>
</tr>
<tr>
<td>Total</td>
<td>g'</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Products</td>
<td>U</td>
<td></td>
<td></td>
<td>Y</td>
<td>q</td>
</tr>
<tr>
<td>Stock changes</td>
<td>-ΔS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supply of residuals</td>
<td>-Wv</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use of residuals</td>
<td>Wu</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resources</td>
<td>R</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emissions</td>
<td>-B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>g'</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1: BalancedPSUTs [17].

**Applications and existing case studies**

Whereas PSUTs are better suited in an accounting perspective, they most often need to be converted into PIOTs to address environmental issues [9]:

- PIOTs can primarily be intended to derive environmental information: environmental pressure indicators, composition of products, element cycles in the economy, dematerialization indicators and physical trade balance indicators;
- PIOTs can be further used for environmental modelling purposes, either to analyze the impact of a certain change in final demand on output (impact analysis) or to impute requirements in raw materials and emissions to a specific final demand (imputation to final demand).

In their literature review [3, 8] list several compiled PIOTs among which: an Austrian PIOTs, for the year 1983, which was the first attempt to calculate a PIOTs[10]; full PIOTs for Germany (for the years 1990 and 1995) [19, 20], for Denmark (1990 updated to 2002) [4, 14] and for New Zealand (1997/98) [13]; an aggregated PIOTs for Italy (1995) [16] and a detailed PIOTs for Finland (1995) [12]. This list can be additionally expanded to the full PIOTs compiled for United
Kingdom (covering the period 1997-2004) [22] and for 22 countries of the European Union (2003) [1].

Despite converting PSUTs to PIOTs is generally necessary, PSUTs may also be directly used for environmental modelling. In particular, PSUTs may be used to forecast future waste quantities, environmental impacts and benefits related to changes in economic activities and policies [18].

Constructing PSUTs involves compiling data which to a larger or smaller extent are inconsistent. A research problem, which has not yet found its final solution, is how to reconcile various sources of information in balancing consistent PSUTs, taking into account all information in the most efficient manner possible. The purpose of the following sections is to present some directions for balancing PSUTs.

Compiling PSUTs: data inventory, uncertainty and inconsistency

In a literature review, [3] report four main methodological differences between existing tables: the level of sector aggregation (from 27 activities in the Danish table to 59 in the German one), the system boundaries, including or not plants and forests, the inclusion or exclusion of different material categories such as water and air, and the base year.

In order to complement this list of methodological differences between existing tables, we performed the review of five studies for which the compilation procedure was sufficiently documented (respectively [1, 13, 14, 20, 22]). From this review, both the data inventory and the treatment of inconsistencies appear as being treated differently from one study to another whereas they are of core importance in the compilation.

PSUTs compilation is mainly driven by the availability of statistical data. The latter generally originate from different sources and are in some cases obtained from rough assumptions in the absence of more accurate information. Focusing on the Supply of products table (V) and on the Use of products table (U), without considering imports/export, emissions, waste, stocks and resources in a first approach, four distinct kinds of data are observed to be usually implemented (Table 1):

1. Sectorial data on the physical supply and use of products, expressed in mass units. These data are directly extracted from national statistical databases. This is the “ideal” case, in the sense that these data can supposedly be directly implemented in the PSUT as such, without any conversion.
2. Sectorial data on the physical supply and use of products, expressed in other units than mass, e.g. in volume or number of items. These data are extracted from national statistical databases and are converted into masses by use of adequate factors.
3. Coefficients of the monetary supply and use tables, as for example annually reported by Eurostat, those need to be converted into physical terms by use of product prices. Import/export commodity prices per net weight may be used as surrogates for their domestic supply/use.
4. Process-specific data, extracted from Life Cycle Inventories and expressing the amount of inputs per unit of output of a specific product. These need to be up-scaled before their implementation into the Use of products table (U).
Table 1: Literature review of data types and sources in V and U compilation

<table>
<thead>
<tr>
<th>Input data for PSUTs (V and U) compilation</th>
<th>Data source</th>
<th>Need for data conversion</th>
<th>Conversion factor</th>
<th>Example of study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical annual data of production and use, per sector. In mass units</td>
<td>National Statistical Institutes</td>
<td>No</td>
<td>N.A.</td>
<td>[1, 14, 20, 22]</td>
</tr>
<tr>
<td>Statistical annual data of use and production, per sector. In units other than mass</td>
<td>National Statistical Institutes</td>
<td>Yes</td>
<td>Mass per unit</td>
<td>[1, 20, 22]</td>
</tr>
<tr>
<td>Monetary Supply Use Table</td>
<td>National Statistical Institutes</td>
<td>Yes</td>
<td>Mass per monetary unit</td>
<td>[1, 13]</td>
</tr>
<tr>
<td>Life Cycle Inventories</td>
<td>Life Cycle Inventories databases (e.g. ecoinvent)</td>
<td>Yes</td>
<td>Upscaling</td>
<td>[1]</td>
</tr>
</tbody>
</table>

PSUTs compilation may require combining several types of data. This is in particular the case for the compilation of the PSUTs of the 22 countries of the European Union for the year 2003 [1]. These PSUTs were compiled by primarily using mass data for products supplied and used, and required as a complement to convert monetary data, process-specific data and number of items into masses in order to cope with missing data. Finally, it is worth reminding that this short review on data types focused on tables V and U (Supply and Use), but that similarly, data from multiple sources are also necessary to compile the tables of imports/exports, emissions, resources, waste generation and use, and stocks.

All the data aforementioned are intrinsically uncertain and convey errors. Uncertainty originates from many sources. For each type of data used in the compilation of Supply and Use tables (V and U), a few causes of uncertainty are reported in the following:

- Physical supply and use data per sector, in mass units, may for example be inaccurate due to errors in reports from enterprises or due to errors in data aggregation;
- Similarly to mass data on products supplied and used per sector, statistical sectorial data on the number of items produced and used may be inaccurate. In addition, factors to convert the number of items into masses are in most cases rough estimates. As an example, converting the number of pairs of shoes annually produced at the scale of a country into a mass (e.g. in tons) requires setting an average mass per pair of shoes. This conversion factor is necessarily inaccurate at the scale of a country production;
- The price per mass of a product category used to convert monetary tables into physical tables may be representative for supplies, but generally fails in representing the different uses of the product (in distinct sectors of the economy). The assumption of homogenous sectorial prices is not valid, as highlighted by discrepancies between monetary and physical input-output model outcomes [21];
- Process-specific data extracted from Life Cycle Inventories databases are generally average data from a small number of plants and processes, for a given country. These data may therefore not be representative at the scale of the production of a country.

The compilation of PSUTs leads to inconsistencies in the mass balancing, as obviously suggested by the large number of uncertainties associated with data and as usually observed in case studies. However, data uncertainty is generally not addressed, whereas inconsistencies are handled manually by modifying coefficients of the Supply-Use Tables (as e.g. performed by [1]). Balancing PSUTs by manual correction are often rather costly to maintain and not easy to document even with the help of modern electronic data processing. Consequently a balancing tool appears necessary to allow the user to build balanced-PSUTs with considering data uncertainties, and therefore making PSUTs more relevant and cost-effective. Such a tool is currently being developed by the writers of the present paper.
BALANCING PSUTS

At the start of balancing an estimate is available for every entry of the PSUTs. In spite of all efforts on compiling real-preliminary estimates, it has to be expected that inconsistencies in the estimates remain. How can inconsistencies be detected and how can they be solved in order to get balanced PSUTs?

In our knowledge, no general theory or useful mathematical programs are available. However, in balancing it is very important to follow a systematic approach to solve the problems. The balancing process is particularly important in the case of detecting and correcting many weaknesses of primary statistics. Moreover, balanced PSUTs can be used for many other purposes than just balancing the national accounts (as mentioned previously). Some experiences [2] show that combination of manual and automatic statistical techniques and procedures is the best workable solution to establish a supply and use system.

Preliminary proposed-strategy: data reconciliation & error estimation

Data reconciliation is a technique that has been developed to improve the accuracy of measurements by reducing the effect of random errors in the data. The principal difference between data reconciliation and other filtering techniques is that data reconciliation explicitly makes use of mass balance identities and obtains estimates of the variables by adjusting measurements so that the estimates satisfy the mass balance constraints [15]. Thereby, data reconciliation improves the accuracy of sectorial national statistical data by adjusting the measured data so that they satisfy the material balance identities.

In general, data reconciliation can be formulated by the following constrained least-squares optimization problem:

\[
\begin{align*}
\text{Min} & & \sum_{i=1}^{n} (y_i - x_i)^2 \\
\text{Subject to} & & g_k(x_i) = 0 \quad k = 1, \ldots, m
\end{align*}
\] (3)

(4)

The objective function (Equation 3) defines the total sum square of adjustment made to measurements; where \( y_i \) is the measurement and \( x_i \) is the reconciled estimate for variable \( i \). Equation 4 defines the set of model constraints (e.g. material mass balance). The deterministic natural laws of conservation of mass (or energy for process engineering) are typically used as constraints for data reconciliation because they are usually known. These types of constraints that are imposed in reconciliation depend on the scope of the reconciliation problem. Furthermore, the complexity of the solution techniques used depends strongly on the constraints imposed. For example, if we are interested in reconciling only the mass flow rates, then the material balances constraints are linear in the mass flow variables and a linear data reconciliation problem results. On the other hand, if we wish to reconcile process data (e.g. temperature or pressure measurements along with flows), then a nonlinear data reconciliation problem occurs. Note that the preliminary proposed-strategy is based on the assumption that only random errors are present in the account measurements which follow a normal (Gaussian) distribution, with zero mean and a known variance-covariance as described in what follows: a tentative PSUTs for the Netherlands.

CASE STUDY

In order to obtain a good understanding of the issues in data reconciliation for future more realistic problems (e.g. full PSUTs compiled for France for the year 2006), a simple case study is introduced here, in order to highlight the assumptions to estimate PSUTs considering data uncertainties. The question is: what is the most efficient way to achieve our objective?
Let us consider the reconciliation of tentative PSUTs. Initially, all mass flow rates are assumed to be known: tables of supply ($V$), use ($U$), imports ($N$), exports ($E$), stocks changes ($\Delta S$), needs fulfillment ($y$), supply and use of residuals ($W_V$ and $W_U$), emissions ($B$) and resources ($R$). The flow measurements contain unknown random errors.

Note that the preliminary proposed strategy is based on the assumption that only random errors are present in the account measurements which follow a normal (Gaussian) distribution, with zero mean and a known variance-covariance. For that reason, the material input and output do not balance. The aim of reconciliation is to make minor adjustments to the measurements in order to make them consistent with the material balances. The adjusted measurements, which are referred to as estimates, are expected to be more accurate than the measurements.

**Tentative PSUTs for the Netherlands 2006**

As an illustration of physical accounting we have used the tentative PSUTs for the Netherlands for 2006 compiled by [8]. According to the author, this is very much a quick-dirty effort to show what the numbers look like for a Western country. It is however also aimed at regaining in-house experience for future research.

Tables 2 & 3 show the PSUTs. The economy has been split into four parts (agriculture, mining, industry and services) which are relevant for material flows. Note that the PSUTs source data is available at about 50-60 industries and many subcategories of wastes, natural resources, etc. The imports and exports of good are part of the Mass Flow Analysis (MFA) statistics produced by the department of environmental accounts. Other data is derived from several accounts such as: air emission, waste, energy and water. The other components, for which physical data is not available, are estimated using the monetary values and appropriate prices from the import and export data [8].

### Table 2: Physical supply table for the Netherlands 2006-millions tons [8]

<table>
<thead>
<tr>
<th>Industries</th>
<th>Agr.</th>
<th>Min.</th>
<th>Ind.</th>
<th>Serv.</th>
<th>Imports</th>
<th>Cons.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agr.</td>
<td>39</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td></td>
<td>63</td>
</tr>
<tr>
<td>Min.</td>
<td>0</td>
<td>113</td>
<td>4</td>
<td>4</td>
<td>157</td>
<td></td>
<td>277</td>
</tr>
<tr>
<td>Ind.</td>
<td>0</td>
<td>0</td>
<td>218</td>
<td>6</td>
<td>144</td>
<td></td>
<td>368</td>
</tr>
<tr>
<td>Serv.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Supply resid.</td>
<td>4</td>
<td>0</td>
<td>48</td>
<td>7</td>
<td>11</td>
<td>9</td>
<td>69</td>
</tr>
<tr>
<td>Emissions</td>
<td>10</td>
<td>3</td>
<td>47</td>
<td>107</td>
<td>0</td>
<td>37</td>
<td>203</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>53</td>
<td>115</td>
<td>317</td>
<td>124</td>
<td>337</td>
<td>46</td>
<td>981</td>
</tr>
</tbody>
</table>

### Table 3: Physical use table for the Netherlands 2006-millions tons [8]

<table>
<thead>
<tr>
<th>Industries</th>
<th>Agr.</th>
<th>Min.</th>
<th>Ind.</th>
<th>Serv.</th>
<th>Cons.</th>
<th>Exp.</th>
<th>Inv.</th>
<th>Final demand</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agr.</td>
<td>2</td>
<td>0</td>
<td>30</td>
<td>1</td>
<td>6</td>
<td>16</td>
<td>0</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>Min.</td>
<td>2</td>
<td>8</td>
<td>210</td>
<td>11</td>
<td>1</td>
<td>80</td>
<td>0</td>
<td>312</td>
<td></td>
</tr>
<tr>
<td>Ind.</td>
<td>12</td>
<td>0</td>
<td>127</td>
<td>28</td>
<td>47</td>
<td>183</td>
<td>4</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>Serv.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Use of resid.</td>
<td>Waste</td>
<td>0</td>
<td>53</td>
<td>12</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>Raw materials</td>
<td>Ores/fuel</td>
<td>175</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>208</td>
<td>5</td>
<td>3652</td>
<td>11179</td>
<td>729</td>
<td>0</td>
<td>0</td>
<td>15773</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>226</td>
<td>189</td>
<td>4072</td>
<td>11230</td>
<td>784</td>
<td>293</td>
<td>4</td>
<td>16608</td>
<td></td>
</tr>
</tbody>
</table>

It is assumed that the flows of all the PSUTs are known and that these measurements contain random errors (they follow a normal distribution, with zero mean and a known variance-covariance). If we denote the true value of the flow rate $i$ by the variable $x_i$, and the corresponding measured value by $y_i$, then we can relate them by the following equation:
\[ y_i = x_i + \varepsilon_i \quad i = 1 \ldots n \] (5)

Where \( \varepsilon_i \) is the random error in measurement \( y_i \). Flows must fulfill the balancing identities presented in Equations 1-2. This means that commodity \((q)\) and industry totals \((g)\) have to be equal in both tables (Figure 1). Obviously the measured values do not satisfy the above equations, since they contain random errors (Tables 2 and 3). It is desired to derive estimates of the flows that satisfy the above flow balances. Intuitively, we can impose the condition that the differences between the measured and estimated flows should be as small as possible (Equation 3). Moreover, we can assume that the error variances for all the measurements are known. Thus, the reconciliation problem is a typically constrained optimization problem with the objective function given by Equation 3 and the constraints given by Equations 1-2. The solution of this optimization problem can be obtained mathematically by means of the preliminary proposed-strategy (being evaluated). It could enhance the benefits of applying data reconciliation techniques. Indeed, by taking into account all the available information about the data, it can prevent from erroneous decisions. A current study aims to validate numerically these assumptions and to extend the strategy to more complex PSUTs (e.g. full PSUTs compiled for France for the year 2006).

**CONCLUSIONS AND RECOMMENDATIONS**

Physical supply-use and input-output tables offer a detailed description of material flows within an economy and in interaction with the environment. These tables can be used either to directly derive environmental information or to perform environmental modelling. However, despite PIOTs and their source PSUTs have clearly appeared as popular in the last decades, as shown by the increasing number of publications and case studies on this issue; their compilation still requires large scale efforts. In addition, despite data estimates generally implemented in PSUTs come from multiple sources and are more or less accurate, errors related to these estimates are generally not accounted for in the PSUTs compilation. Accordingly there is a strong need to limit the cost and time required for PSUTs and PIOTs compilation while at the same improving their reliability. The strategy of data reconciliation & error estimation proposed to reconcile tables would benefit greatly by addressing these two core issues. First it would significantly reduce the time necessary to produce PSUTs. Secondly, it would also improve the accuracy of data by adjusting the measured values so that they satisfy the process constraints.

**Bibliography**