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# 1 Shallow-structure characterization by 2D elastic full-waveform inversion

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3 Stephane Operto<sup>5</sup>, and Jean Virieux<sup>3</sup>

## 4 ABSTRACT

26 Assessing the effectiveness of elastic full-waveform-inversion  
27 (FWI) algorithms when applied to shallow 2D structures  
28 in the presence of a complex topography is critically impor-  
29 tant. By using FWI, we overcome inherent limitations of con-  
30 ventional seismic methods used for near-surface prospecting  
31 (acoustic tomography and multichannel spectral analysis of  
32 surface waves). The elastic forward problem, formulated in  
33 the frequency domain, is based on a mixed finite-element P0-  
34 P1 discontinuous Galerkin method to ensure accurate model-  
35 ing of complex topography effects at a reasonable computing  
36 cost. The inversion problem uses an FWI algorithm to mini-  
37 mize the misfit between observed and calculated data. Based  
38 on results from a numerical experiment performed on a realis-  
39 tic landslide model inspired from the morphostructure of the  
40 Super-Sauze earthflow, we analyzed the effect of using a hier-  
41 archical preconditioning strategy, based on a simultaneous

multifrequency inversion of damped data, to mitigate the  
strong nonlinearities coming from the surface waves. This  
strategy is a key point in alleviating the strong near-surface  
effects and avoiding convergence toward a local minimum.  
Using a limited-memory quasi-Newton method improved the  
convergence level. These findings are analogous to recent  
applications on large-scale domains, although limited source-  
receiver offset ranges, low-frequency content of the source,  
and domination of surface waves on the signal led to some dif-  
ficulties. Regarding the impact of data decimation on the  
inversion results, we have learned that an inversion restricted  
to the vertical data component can be successful without sig-  
nificant loss in terms of parameter imagery resolution. In our  
investigations of the effect of increased source spacing, we  
found that a sampling of 4 ms (less than three times the theo-  
retical maximum of one half-wavelength) led to severe  
aliasing.

## 41 INTRODUCTION

42 Accurate subsurface imaging based on seismic methods con-  
43 stitutes one of the main issues encountered in the environmental  
44 and civil engineering fields. It offers the possibility of taking  
45 advantage of a noninvasive technique to depict subsoil structures  
46 of the first 100 m as a reconstructed image from a seismic  
47 wavefield recorded at the surface. This can be achieved using  
48 several reconstruction techniques that analyze different kinds of  
49 waves associated with propagation phenomena (diffraction,  
50 reflection, dispersion, refraction, etc.).

The most conventional technique is based on the inversion of  
body-wave arrival traveltimes, particularly P-waves, using direct  
or refracted waves. Efficiency of the process closely depends on  
the realism of the associated forward problem to account for the  
characteristics of the medium (heterogeneities, contrasts) in cal-  
culating traveltimes. In this context, robustness and efficiency of  
the ray-tracing technique, based on the asymptotic ray theory in  
the high-frequency approximation, are restricted to the case of  
smoothed media (Červený et al., 1977; Červený, 2001) and con-  
sequently are unsuitable for highly heterogeneous subsurface  
domains.

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62 An alternative to ray tracing and the more robust wavefront  
63 construction technique (Vinje et al., 1993, 1996a, 1996b) con-  
64 sists of applying finite differences to solve the eikonal equation  
65 numerically (Vidale, 1988; Podvin and Lecomte, 1991), making  
66 it possible to deal with more heterogenous media. Important  
67 progress has been achieved to handle the associated inverse  
68 problem efficiently, using the popular simultaneous iterative  
69 reconstruction technique (SIRT) (van der Sluis and van der  
70 Vorst, 1987; Grandjean and Sage, 2004) or the more appealing  
71 adjoint state method (Taillandier et al., 2009). However, appli-  
72 cations to real data in the context of shallow prospecting  
73 (Grandjean and Leparoux, 2004; Ellefsen, 2009) reveal restric-  
74 tions. This is particularly the case when later arrivals must be  
75 included in the inversion scheme or when dealing with real data  
76 where surface waves, which always represent the main compo-  
77 nent (about two-thirds) of the seismic energy, can seriously miti-  
78 gate signals used in the inversion.

79 On the other hand, with the introduction of the spectral analy-  
80 sis of surface waves (SASW) method (Nazarian and Stokoe,  
81 1984, 1986; Stokoe and Nazarian, 1985; Stokoe et al., 1988),  
82 surface waves have received much attention. The good signal-  
83 to-noise ratio (S/N) of these waves associated with the relative  
84 ease of their acquisition gives rise to a variety of applications  
85 (Lai, 1998; Park et al., 1999; Rix et al., 2001). Early studies  
86 were devoted to reconstructing 1D shear-wave velocity distribu-  
87 tion by calculating phase differences between two receivers. The  
88 SASW method was later extended to the multichannel analysis  
89 of surface waves (MASW), which is based on the phase-velocity  
90 variation with frequency from a multichannel recording system.

91 The 1D assumption of the MASW method is imposed by the  
92 formulation used for solving the inverse problem (Hermann,  
93 1991). To overcome this limitation, some extensions have been  
94 made (Park et al., 1998; Xia et al., 1999; Grandjean and Bitri,  
95 2006) to adapt the methodology to 2D contexts by narrowing  
96 offset windows or/and using a summation principle to increase  
97 the S/N. The resulting 1D velocity profiles are then interpolated  
98 along a seismic line to produce a 2D view of the shear velocity.

99 Despite all of these developments, some limitations still alter  
100 the potential of surface-wave methods. These limitations are  
101 mainly the result of difficulties encountered when identifying  
102 and separating the first (fundamental) propagation mode from  
103 higher modes (possible propagation modes of surface waves in a  
104 layered medium), which form the basis of the inversion process.  
105 This phenomenon, in addition to the errors resulting from fitting  
106 data including 2D or 3D effects (phase-velocity changes) under  
107 a 1D assumption, drastically mitigate the efficiency of the  
108 MASW method (Bodet, 2005). Some recent results have also  
109 shown that the dispersion curve is not an intrinsic property of  
110 the medium by emphasizing the influence of acquisition param-  
111 eters (Socco and Strobina, 2004).

112 An important point is the common feature of MASW and  
113 first-break acoustic tomography. Both use a restrictive part of  
114 the information contained in the seismic signal: the dispersion  
115 of Rayleigh waves and the first P-wave arrivals. A strategy inte-  
116 grating both signals should be more efficient and physically con-  
117 sistent to reduce the possible solutions satisfying the approaches.  
118 To overcome this issue, an alternative approach consists of tak-  
119 ing advantage of recent advances in quantitative imaging based  
120 on full-waveform inversion (FWI) in the time (Tarantola, 1984)  
121 or frequency domains (Pratt et al., 1998). In theory, these

approaches offer important possibilities because they use all in-  
formation contained in seismic signals (P-waves in the acoustic  
case and P-SV-SH-waves in the elastic case) in the inversion  
strategy. The inverse problem formulation in the frequency do-  
main has been implemented and applied to synthetic and real  
data concerning large-scale domains (kilometric scale) (Ravaut  
et al., 2004; Brenders and Pratt, 2007; Brossier et al., 2009).  
This context is very different from the subsurface one because  
low-frequency sources and long offsets can be used. Surface  
waves can be separated easily from body waves and tradition-  
ally are muted.

However, in the context of near-surface imaging, the greatest  
part of the energy emitted by a surface seismic source contrib-  
utes to the generation of surface waves. To overcome this limi-  
tation, some workers have proposed applying a time window to  
the early arrivals and performing acoustic waveform tomogra-  
phy with near-surface data (Gao et al., 2006, 2007; Sheng et al.,  
2006; Smithyman et al., 2009). Results show that this strategy  
outperforms travelt ime tomography and is well suited to data  
coming from refraction surveys where far and intermediate off-  
sets are considered. However, when the offset range is too small  
(which is usually the case in near-surface prospecting) to allow  
separation between body waves and surface waves, the effi-  
ciency of this strategy may be severely altered. Moreover, this  
strategy does not take advantage of the information included in  
shear and surface waves that are usually considered as a source  
of noise in the inversion. These waves propagate with a lower  
velocity than compressional waves and may therefore lead to  
higher resolution of the images.

To our knowledge, only a few studies of waveform inversion  
involving body and surface waves have been performed for  
near-surface (0–100 m) investigations (Gélis et al., 2007; Romd-  
hane et al., 2008). These numerical investigations were per-  
formed to image near-surface heterogeneities with various con-  
trasts in a well-known background medium. In these cases,  
surface topography was considered to be flat, mainly because of  
the computational difficulties encountered to model surface  
waves accurately in the presence of a complex topography  
(Moczo et al., 2007). The effects of irregular topography on  
seismic wave motion have been the subject of some numerical  
investigations (Bleibinhaus and Rondenay, 2009; Shiann-Jong  
et al., 2009). It is well established that topography can drasti-  
cally influence amplitudes and phases of the seismic signal.  
Consequently, correct modeling of free-surface effects is a criti-  
cal requirement for any seismic-inversion process.

Recent work conducted by Brossier et al. (2009) focuses on  
the impact of applying several multiresolution strategies to miti-  
gate the strong nonlinearity inherent in surface waves. Simula-  
tions performed with a section of the well-known SEG/EAGE  
overthrust model reveal that preconditioning provided by time  
damping associated with successive inversions of overlapping  
frequency groups is critical to converge toward acceptable ve-  
locity models.

The objective of our work is thus to evaluate the effectiveness  
of using an FWI algorithm to take advantage of the information  
contained in surface waves to image heterogeneous shallow  
structures in the context of a complex surface topography. Our  
paper is organized in two sections. In the first section, we pres-  
ent a brief review of the basis of the elastic FWI technique  
used. In the second section, we apply it to a synthetic but

182 realistic landslide case, derived from the structure of the Super-  
183 Sauze earthflow. We evaluate the efficiency of using precondition-  
184 ing strategies to reconstruct the shallow velocity structure.  
185 We also address the effects of some practical considerations,  
186 particularly the restriction to the vertical data component and  
187 the impact of acquisition decimation, typically related to subsur-  
188 face prospecting, on inversion results.

## 189 ELASTIC FULL-WAVEFORM INVERSION

190 We first consider the 2D P-SV-wave modeling case. The for-  
191 ward and inverse problems are solved in the frequency domain.  
192 The forward problem is based on a discontinuous Galerkin (DG)  
193 approach. An FWI algorithm is used to solve the inverse prob-  
194 lem. It is based on the preconditioned conjugate-gradient (PCG)  
195 method or a limited-memory quasi-Newton Broyden-Fletcher-  
196 Goldfarb-Shanno (L-BFGS) approach and is implemented on a  
197 parallel computation architecture. We then present the precondition-  
198 ing strategies used for our numerical simulations.

### 199 The forward problem

200 In an isotropic elastic medium, the equation system governing  
201 the wave propagation in 2D media relates velocities  $\partial V_x$  and  $V_z$   
202 to stresses  $\sigma_{xx}$ ,  $\sigma_{zz}$ , and  $\sigma_{xz}$ . It can be written in the frequency  
203 domain as

$$\begin{aligned} -i\omega\rho V_x &= \frac{1}{\rho(\mathbf{x})} \left[ \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xz}}{\partial z} \right] + F_x, \\ -i\omega\rho V_z &= \frac{1}{\rho(\mathbf{x})} \left[ \frac{\partial\sigma_{xz}}{\partial x} + \frac{\partial\sigma_{zz}}{\partial z} \right] + F_z, \\ -i\omega\sigma_{xx} &= (\lambda(\mathbf{x}) + 2\mu(\mathbf{x})) \frac{\partial V_x}{\partial x} + \lambda(\mathbf{x}) \frac{\partial V_z}{\partial z}, \\ -i\omega\sigma_{zz} &= \lambda(\mathbf{x}) \frac{\partial V_x}{\partial x} + (\lambda(\mathbf{x}) + 2\mu(\mathbf{x})) \frac{\partial V_z}{\partial z}, \\ -i\omega\sigma_{xz} &= \mu(\mathbf{x}) \left[ \frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right], \end{aligned} \quad (1)$$

204 where  $\lambda$  and  $\mu$  are the Lamé coefficients,  $\mathbf{x}$  is spatial position,  $\rho$   
205 is density, and  $\omega$  is angular frequency. The physical properties  
206 of the medium are supposed to be constant inside each cell, and  
207 central numerical fluxes are used. Details of the mixed DG  
208 interpolation orders P0-P1 formulation, used in this study, are  
209 provided in [Brossier \(2009\)](#).

210 System 1 can be written with respect to a linear matrix for-  
211 malism for each frequency considered:

$$212 \quad \mathbf{A}\mathbf{x} = \mathbf{s} \quad (2)$$

213 where vector  $\mathbf{x}$  denotes the unknowns, consisting of the particle  
214 velocities and stresses,  $\mathbf{s}$  is the source term, and  $\mathbf{A}$  is the imped-  
215 ance matrix. To solve the linear system resulting from discretizing  
216 equation 2, the impedance matrix is first factorized with an LU  
217 decomposition independent of the source term. Solutions for multi-  
218 ple sources (i.e., multiple right-hand-side terms) can then be  
219 obtained efficiently by forward and backward substitutions. Parallel  
220 factorization of the impedance matrix is performed using the  
221 MUMPS massively parallel direct solver package ([MUMPS, 2009](#)).

222 Some recent results reveal promising prospects for applying  
the DG method to elastic-wave propagation. The use of high

orders of interpolation is especially appealing because they  
allow unstructured meshes and thus offer the possibility of  
locally adapting the mesh size to local medium parameters (h-  
adaptive mesh). They also ensure high accuracy with a coarse  
discretization of the medium ([Dumbser and Käser, 2006](#)). How-  
ever, this coarse discretization may be inconsistent with the  
expected resolution of the FWI, which necessitates a discretiza-  
tion close to  $\lambda/4$  ([Soubrier et al., 2009](#)).

In this study, we use a lower order of interpolation. Applying  
the DG method based on the lowest interpolation order (P0)  
turns out to be very efficient, in terms of computational cost, in  
comparison with classical finite-difference formulations in the  
context of contrasted media and smooth surface topography  
([Brossier et al., 2008](#)). The accuracy is guaranteed with only  
10–15 cells per minimum wavelength compared to the 60 grid  
points necessary with the rotated second-order stencil and the  
vacuum formalism ([Saenger et al., 2000](#); [Bohlen and Saenger, 2006](#)).  
In addition, an interesting compromise between accuracy  
of wavefield estimation and computational cost consists of using  
the mixed P0-P1 DG interpolation to overcome some particular  
restrictions related to topography complexity. The use of  
unstructured meshes (for P1) ensures precise implementation of  
the source term and accurate modeling of the complex topogra-  
phy, taking into account the free-surface boundary conditions  
([Brossier, 2010](#)). In addition, it offers the possibility to adapt  
mesh size to the local physical parameters. This property is of  
great interest, especially in the context of near-surface modeling  
with weathered zones (with very low velocities).

### The inverse problem

In this section, we briefly review the principles of FWI. An  
extensive overview of the method can be found in [Virieux and  
Operto \(2009\)](#).

In the case of weighted least-squares linearized inversion, the  
misfit function  $E$  can be expressed ([Tarantola, 1987](#)) as

$$E(\mathbf{m}) = \frac{1}{2} (\mathbf{d}_{\text{obs}} - \mathbf{d}_{\text{cal}})^\dagger \mathbf{S}_d^\dagger \mathbf{S}_d (\mathbf{d}_{\text{obs}} - \mathbf{d}_{\text{cal}}), \quad (3)$$

where the dagger  $\dagger$  denotes the transpose conjugate;  $\mathbf{d}_{\text{obs}}$  and  
 $\mathbf{d}_{\text{cal}}$  denote observed and calculated data (particle velocities),  
respectively;  $\Delta\mathbf{d} = (\mathbf{d}_{\text{obs}} - \mathbf{d}_{\text{cal}})$  corresponds to the data-misfit  
vector in model  $\mathbf{m}$ ; and  $\mathbf{S}_d$  is a weighting operator applied to  
the data. A solution to equation 3 is to linearize it in the second  
order around an initial model  $m_l$ , which corresponds to the  
model of the  $l$ th iteration as follows:

$$E(\mathbf{m}^{(l-1)} + \delta\mathbf{m}^{(l)}) = E(\mathbf{m}^{(l-1)}) + \nabla_m E(\mathbf{m}^{(l-1)}) \delta\mathbf{m}^{(l)} \\ + \frac{1}{2} \delta\mathbf{m}^{(l)\dagger} H(\mathbf{m}^{(l-1)}) \delta\mathbf{m}^{(l)}, \quad (4)$$

where  $\delta\mathbf{m}^{(l)}$  is the model perturbation and where  $\nabla_m E(\mathbf{m}^{(l-1)})$   
and  $H(\mathbf{m}^{(l-1)})$  are the gradient and Hessian of the misfit function,  
respectively. Minimizing  $E$  leads to the Newton equation, which  
relates the model perturbation to the gradient and Hessian as

$$\delta\mathbf{m}^{(l)} = - \left[ H(\mathbf{m}^{(l-1)}) \right]^{-1} \nabla_m E(\mathbf{m}^{(l-1)}). \quad (5)$$

The gradient direction is computed efficiently following the  
adjoint-state formulation ([Plessix, 2006](#)). For one model param-  
eter  $k$ , the system can be recast in matrix form:

$$\nabla_m E(m_k) = \Re \left[ \mathbf{x}^t \left[ \frac{\delta \mathbf{A}}{\delta m_k} \right]^t \mathbf{A}^{-1} \tilde{\mathcal{P}} \mathbf{S}_d^\dagger \mathbf{S}_d \mathbf{d}^* \right], \quad (6)$$

where  $\tilde{\mathcal{P}}$  is an operator that projects the data residual vector in the data space to the model space,  $\Re$  is the real part of a complex number,  $\Delta \mathbf{d}$  corresponds to the data misfit vector, and  $t$  and  $*$  are the transpose and conjugate operators. Equation 6 shows that the gradient is formed by a weighted product of the incident wavefield  $\mathbf{x}$  and the adjoint wavefield  $\mathbf{A}^{-1} \tilde{\mathcal{P}} \mathbf{S}_d^\dagger \mathbf{S}_d \mathbf{d}^*$ . The gradient of the misfit function therefore requires computing only two forward problems per shot.

In practice, for realistically sized problems, resolving the Newton equation (equation 6) is avoided because of the large inherent cost. One alternative used in this study consists of considering only diagonal terms of the Hessian or the pseudo-Hessian matrix (Pratt et al., 1998; Shin et al., 2001) as a preconditioner for the optimization algorithm.

To overcome the diagonal estimation of the Hessian, an L-BFGS method can be used (Nocedal and Wright, 1999). This algorithm is more efficient than the preconditioned nonlinear conjugate gradient for solving FWI problems (Brossier et al., 2009). The algorithm estimates a nondiagonal inverse Hessian from the  $m$  most recent gradient and model vectors. An initial estimate of the Hessian can be provided from the diagonal terms of an approximate Hessian. An example of the contribution of this method to improve the convergence level of the misfit function is illustrated later.

Once the right-hand side of equation 6 is estimated, the model is updated iteratively:

$$\mathbf{m}^l = \mathbf{m}^{(l-1)} + \alpha \delta \mathbf{m}^{(l)}, \quad (7)$$

where  $\alpha^{l-1}$  denotes the step length, estimated in this study by parabola fitting.

### Efficient mitigation of nonlinear effects

FWI is carried out by proceeding iteratively from low to high frequencies. This allows short wavelengths to be introduced progressively in the parameter images and thus helps to mitigate the nonlinearity of the inverse problem. The strategy has proven effective for the acoustic inverse problem (Pratt, 1999; Ravaut et al., 2004; Operto et al., 2006). In the elastic case, work conducted to evaluate the ability of the method to locate small heterogeneities in shallow subsurface structures in the presence of a flat topography (Gélis, 2005) reports many difficulties stemming from the presence of complex wave phenomena, particularly surface waves. Because the waves contain most of the seismic energy and because they interact strongly with the topographic irregularities, we speculate that they will significantly govern the optimization process and constraint the algorithm to explore a wrong solution and reach a local minimum.

To fulfill our objective, i.e., imaging shallow and highly contrasted velocity structures in the presence of a complex topography, we must take into consideration three critical points.

- 1) An accurate starting model is required. It must be close enough to the true velocity model to avoid the cycle-skipping phenomenon, which may occur when the error traveltime is greater than half a period.
- 2) The receiver antenna must be long enough to ensure good model illumination. Limited-aperture acquisition geometries can result in the algorithm being trapped in a local minimum.

- 3) The choice of inverted frequencies is critical to guarantee accurate coverage in terms of long and short wavelengths, especially for the S-wave velocity parameter  $V_S$ . Low frequencies must be considered to avoid convergence toward a local minimum at an early stage. This restriction also explains the necessity of considering a starting model close enough to the real one. Moreover, selection of the inverted frequencies must ensure a continuous wavenumber illumination following, for example, the strategy proposed by Sirgue and Pratt (2004).

An alternative to mitigate the strong nonlinearities resulting from complex wave phenomena consists of defining two levels of hierarchy (Brossier et al., 2009). The first is to perform successive inversions of overlapping groups of finite frequencies to better constrain the algorithm and take into account the redundant information contained in the selected frequencies. Frequencies of each group are inverted simultaneously, and the overlapping (frequencies in common) between two successive groups is minimized. Application of this strategy to the SEG/EAGE overthrust model reveals some improvements in comparison to a sequential single-frequency approach.

The second level consists of progressively introducing later arrivals (converted waves, surface waves) in the inversion. In the time domain, this level of hierarchy can be implemented in a flexible way by time windowing (Pratt and Shipp, 1999). In the frequency domain, only time damping can be used (Shin et al., 2002). Time damping of a seismic signal  $x(t)$  with respect to the first-arrival traveltime  $t_0$ , for example, can be implemented in the frequency domain by introducing a complex-valued frequency following the expression

$$X(w + i\gamma) \exp(\gamma t_0) = \int_{-\infty}^{+\infty} x(t) \exp(-\gamma(t - t_0)) \exp(-iwt) dt, \quad (8)$$

where  $\gamma$  denotes the applied damping factor.

To assess the effectiveness of these strategies in our context, we conducted a numerical study for a realistic landslide model. For all tests presented hereafter, the inverted model parameters are P- and S-wave velocities. The source-parameter estimate is not addressed, although it is a critical issue when applying FWI to real data. The proposed numerical tests are performed to highlight two aspects. In the first section, we evaluate the performance of the defined preconditioning strategies to recover the velocity structures and to assess the contribution of the L-BFGS optimization method. In the second section, we study the effect of decimating the acquisition geometry on the inversion results, notably in term of number of sources, to be as close as possible to realistic cases.

## LANDSLIDE SYNTHETIC CASE STUDY: A NUMERICAL EXPERIMENT

The landslide model was inspired from a transverse section of the Super-Sauze earthflow located in the French Alps (Flageollet et al., 2000). It consists of a  $210 \times 60$ -m section composed of several velocity layers, as proposed by Grandjean et al. (2006), after performing first-arrival tomography. The medium is

376 characterized by strong lateral velocity variations associated  
 377 with highly contrasted media, with P- and S-wave velocities  
 378 varying from 800 to 3200 m/s and 480 to 1600 m/s, respectively  
 379 (Figure 1; Table 1) with an inconstant Poisson's ratio. Here we  
 380 used a constant density of 1600 g/cm<sup>3</sup> for forward and inverse  
 381 problems. The surface topography is highly irregular.

382 Simulations were performed using a Ricker source wavelet  
 383 with a peak frequency of 60 Hz. In a real data context, the  
 384 source signature and radiation pattern are additional unknowns  
 385 that can be estimated by solving a linear inverse problem (Pratt,  
 386 1999; Virieux and Operto, 2009). The chosen parameters corre-  
 387 spond to wavelengths (at the central frequency) varying between  
 388 53.3 and 13.4 m for  $V_P$  and between 26.6 and 8 m for S-wave  
 389 velocity  $V_S$ . The frequency bandwidth covers the interval  
 390 [10,150] Hz. Detecting shallow structures of metric scale is thus  
 391 affected by the bad resolution of the thin layers of the model.  
 392 From a numerical point of view, a source with higher-frequency  
 393 content should overcome this limitation. However, this assump-  
 394 tion is meaningless in practice because high-frequency signals  
 395 are strongly attenuated in the shallow, fissured layers of the me-  
 396 dium and thus useless.

397 The mesh is divided into a 1-m-thick unstructured layer for  
 398 P1 interpolation and a structured layer (made of equilateral tri-  
 399 angles) for P0. This choice ensures at least 15 grid cells per  
 400 minimum propagated wavelength (corresponding to the surface  
 401 wave estimated from the shear wave and the Poisson's ratio  
 402 with the Viktorov formula [Viktorov, 1965]) for the highest  
 403 modeled frequency.

404 A total of 197 explosive (Ricker) sources were considered 1  
 405 m below the surface, with a 1-m spacing along the horizontal  
 406 axis; 197 receivers were located 0.5 m below the surface. Verti-  
 407 cal and horizontal particle velocities were computed. An exam-  
 408 ple of vertical and horizontal components of one shot gather  
 409 (Figure 2) shows that most of the seismic energy is radiated in  
 410 the form of surface waves. It also highlights the footprint of the  
 411 irregular topography on the seismic signal, which is drastically  
 412 warped. Strong diffractions resulting from the topography shape  
 413 can be observed, particularly for the incident surface waves.

414 **Impact of inversion conditioning**

415 In our tests,  $V_P$  and  $V_S$  are the inverted model parameters,  
 416 and density is supposed to be known. Starting models are  
 417 smoothed versions of true ones, obtained after applying a 2D  
 418 Gaussian smoothing function with a spatial correlation length of  
 419 6 m for  $V_S$  (Figure 3) and  $V_P$  parameters. This constitutes a

420 good compromise between severely altering the delineation of  
 421 layer interfaces and avoiding the cycle-skipping phenomenon  
 422 that may occur when the starting models are too far from the  
 423 real ones. For the shallow layers, it is a realistic model that can  
 424 be obtained by conventional methods. These models also sup-  
 425 pose that we have a priori knowledge on the shape of the bed-  
 426 rock. Examples of vertical and horizontal components of shot  
 427 gathers are shown in Figures 4a and 4b, for comparison with  
 428 those of Figures 2a and 2b.

429 *Successive inversions of single frequencies*

430 In a first step, sequential inversion is performed with respect  
 431 to the selected frequencies of Table 2 to ensure a continuous  
 432 wavenumber illumination (Sirgue and Pratt, 2004). Horizontal  
 433 and vertical components are considered. A maximum of 25 iter-  
 434 ations per frequency is performed to ensure convergence of the  
 435 algorithm at reasonable computing cost.

436 The final models obtained are shown in Figures 5a and 6.  
 437 The inversion fails to converge toward an acceptable model for  
 438  $V_P$  and  $V_S$ . Indeed, the main features of the layered structure are  
 439 not recovered. In addition, strong artifacts are observed. The  
 440 algorithm has converged into a local minimum because we  
 441 observe strong, unrealistic anomalies near the free surface  
 442 (Figure 5b and Figure 7b). This failure can be attributed to the  
 443 dominant contribution of surface waves that prevents the high-  
 444 frequency signals associated with body waves to be considered  
 445 in the inversion. Similar effects have been observed by Gélis  
 446 et al. (2007) and Romdhane et al. (2009).

447 *Frequency group inversion of damped data*

448 In this section, we investigate the performance of a simultane-  
 449 ous inversion of damped data. We consider three overlapping  
 450 groups of three frequencies (see Table 2), with damping

**Table 1. Maximum and minimum velocity parameters for the landslide model. Maximum and minimum wavelengths are calculated for the lowest and highest inverted frequencies, respectively.**

$V_P$ (m/s)	$V_S$ (m/s)	$\lambda_{V_{Pmax}}$ (m)	$\lambda_{V_{Pmin}}$ (m)	$\lambda_{V_{Smax}}$ (m)	$\lambda_{V_{Smin}}$ (m)
800	480	23.8	5.9	11.9	3.57
3200	1600	150.2	37.6	75.1	22.5

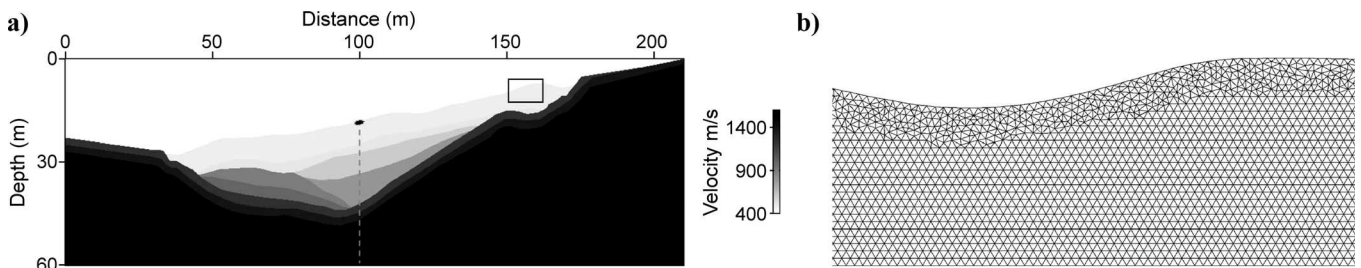


Figure 1. (a)  $V_S$  true velocity model for the (realistic) landslide model. The gray dashed line and the black star correspond to the position of the extracted vertical profiles and the source position of shot gathers depicted in the following figures, respectively. (b) Zoom of the boxed area, showing the mesh used for the landslide model with the mixed DG P0-P1 method.

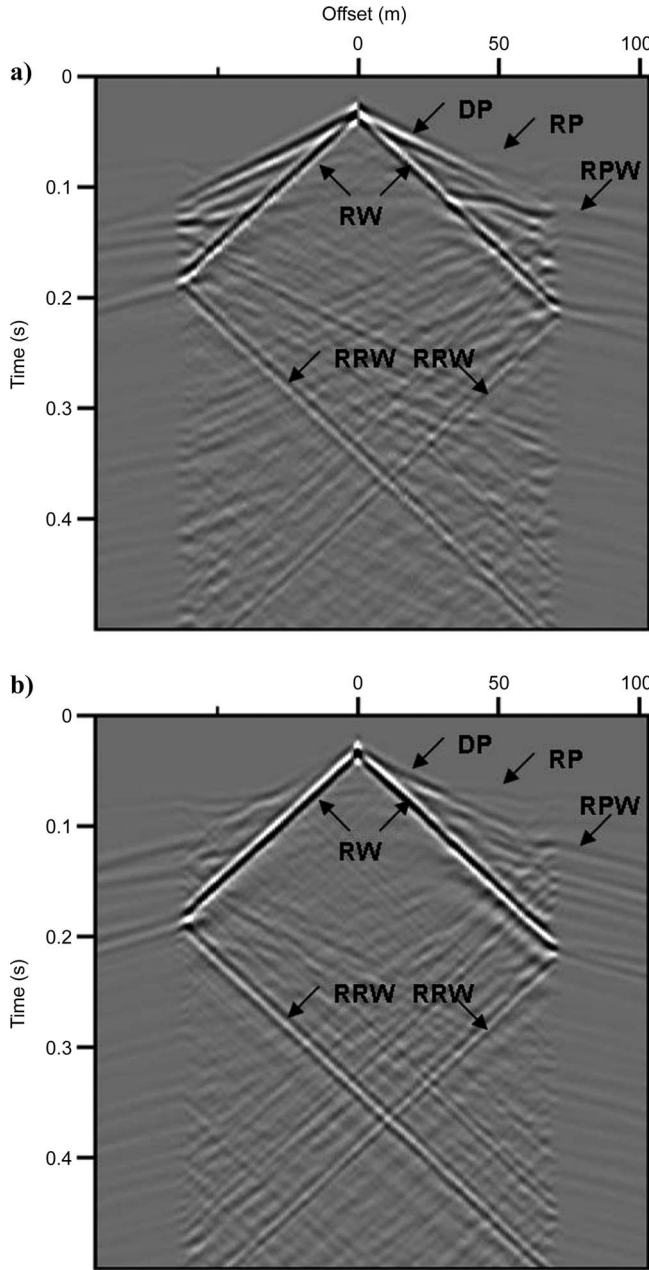


Figure 2. Examples of (a) horizontal and (b) vertical synthetic shot gathers of the landslide model. The shot position correspond to an abscissa of 100 m on the horizontal distance axis of Figure 1. DP, RP, and RPW correspond to direct, refracted, and reflected P-waves, respectively. RW and RRW correspond to Rayleigh waves (fundamental mode) and back-propagated Rayleigh waves.

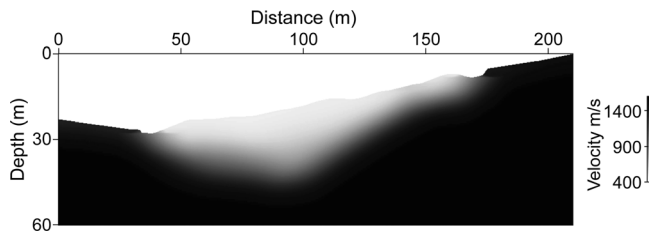


Figure 3. Starting  $V_S$  model considered for the landslide case.

coefficients varying between 20 and 1.5. Figures 4c and 4d shows the vertical and horizontal components obtained with a damping coefficient of 20. The shot position used in Figure 2 is considered. Comparison of shot gathers of Figures 2 and 4 highlights the role of data damping to progressively introduce surface waves as well as complex free-surface reflections, particularly for the far offsets. A maximum of 25 iterations was performed for each damped frequency group.

Final results, obtained after inverting the three groups (Figures 8 and 9), reveal how crucial this strategy is to converge successfully toward an acceptable solution. For  $V_S$ , focusing the shallower structures is defined with a high level of resolution. For the deeper layers, the model is slightly improved and the contribution is less significant as the velocity values (and thus

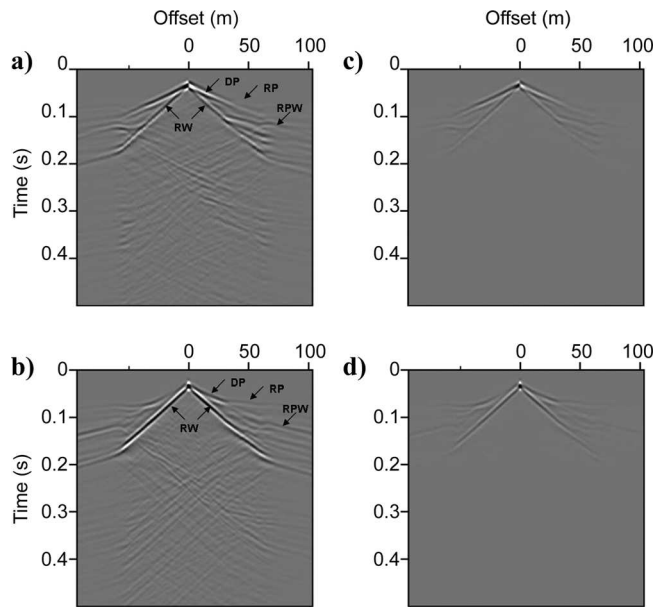


Figure 4. (a) Horizontal and (b) vertical synthetic shot gathers of the starting landslide model. (c) Damped horizontal and (d) vertical synthetic shot gathers of the starting landslide model ( $\gamma = 20$ ). The shot position correspond to an abscissa of 100 m on the horizontal distance axis of Figure 1. DP, RP, and RPW correspond to direct, refracted, and reflected P-waves, respectively. RW correspond to Rayleigh waves (fundamental mode).

Table 2. Sequential inverted frequencies, frequency groups, and damping coefficients considered for the landslide model.

Frequency group	Sequential frequency (Hz)	Simultaneous frequency (Hz)	Damping coefficient (1/s)
1	21.3	21.3, 27.5, 42.7	20, 5, 1.51
2	27.5	42.7, 61.0, 82.4	
3	42.5	82.4, 106.8, 134.3	
4	61.0		
5	82.4		
6	106.8		
7	134.3		

465 the associated wavelengths) increase (Figures 8 and 10b). The  
 466 weak contribution of the inversion process to reconstruct  $V_P$   
 467 (Figures 9 and 10a) was expected and can be explained by the  
 468 lack of short wavelengths illuminated with respect to the  $V_P$   
 469 model velocities (see the wavelengths associated with  $V_P$  in  
 470 Table 2). Few artifacts can, however, be noticed in the zones  
 471 corresponding to the highest velocity contrasts (between the  
 472 shallowest layer and the bedrock, in Figure 8b).

473 Examples of an initial differential seismogram (difference  
 474 between data calculated with the true model and data calculated  
 475 with the starting model) and a final differential seismogram (dif-  
 476 ference between data calculated in the final model and data cal-  
 477 culated with the true model) are depicted in Figures 10c and 10d,  
 478 respectively. The comparison shows that the unexplained energy  
 479 mainly comes from the back-propagated Rayleigh waves at the  
 480 highest velocity contrasts.

481 The penetration depth of the Rayleigh wave is approximately  
 482 half of its wavelength. This means it will dominate the low-fre-  
 483 quency part of the data spectrum, whereas body waves will  
 484 dominate the high-frequency part of the spectrum. Applying  
 485 strong damping coefficients to the high frequencies to favor the  
 486 use of body waves is therefore unnecessary, in our opinion.

487 We have also investigated the effect of resampling the fre-  
 488 quency interval in the inversion group. We have divided the fre-  
 489 quency interval by a factor of two and considered three groups  
 490 of five (instead of three) frequencies. This resampling is

491 expected to strengthen the spectral redundancy and yield a  
 492 higher definition of layers. The same damping coefficients were  
 493 considered as in the previous example. Results in Figure 11  
 494 show a slight improvement of  $V_S$  parameter reconstruction at  
 495 the expense of a significantly higher computing cost.

Contribution of L-BFGS method

496 We performed an inversion test using the L-BFGS optimiza-  
 497 tion method with the same frequency groups and damping coef-  
 498 ficients as in the previous section (with respect to Table 2). The  
 499 initial estimate of the pseudo-Hessian is provided by the diagonal ele-  
 500 ments of the pseudo-Hessian (Shin et al., 2001), and five differ-  
 501 ences of cost-function gradients and model vectors are used for  
 502 the L-BFGS algorithm. Figures 12a and 12b and Figures 13a  
 503 and 13b show final  $V_P$  and  $V_S$  reconstructed velocity models and  
 504 vertical extracted profiles for each parameter, respectively. 505

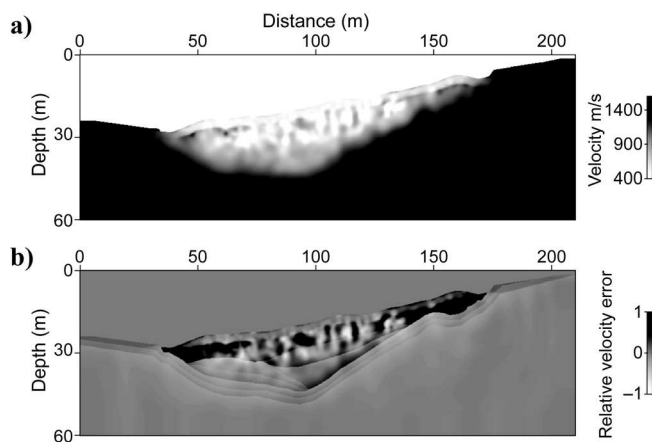


Figure 5. (a) Final  $V_S$  model and (b) relative velocity error (ratio of the velocity error to the true velocity), obtained after sequential inversion of seven frequencies varying from 21.3–134.3 Hz as indicated in Table 2. Vertical and horizontal components are inverted.

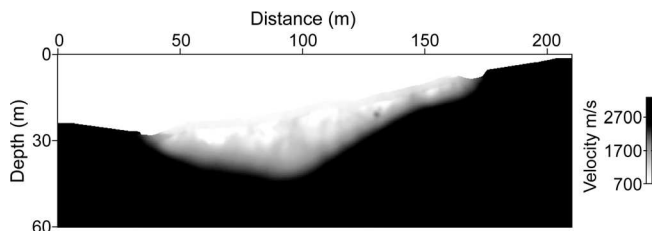


Figure 6. Final  $V_P$  model obtained after sequential inversion of seven frequencies varying from 21.3 to 134.3 Hz as indicated in Table 2. Vertical and horizontal components are inverted.

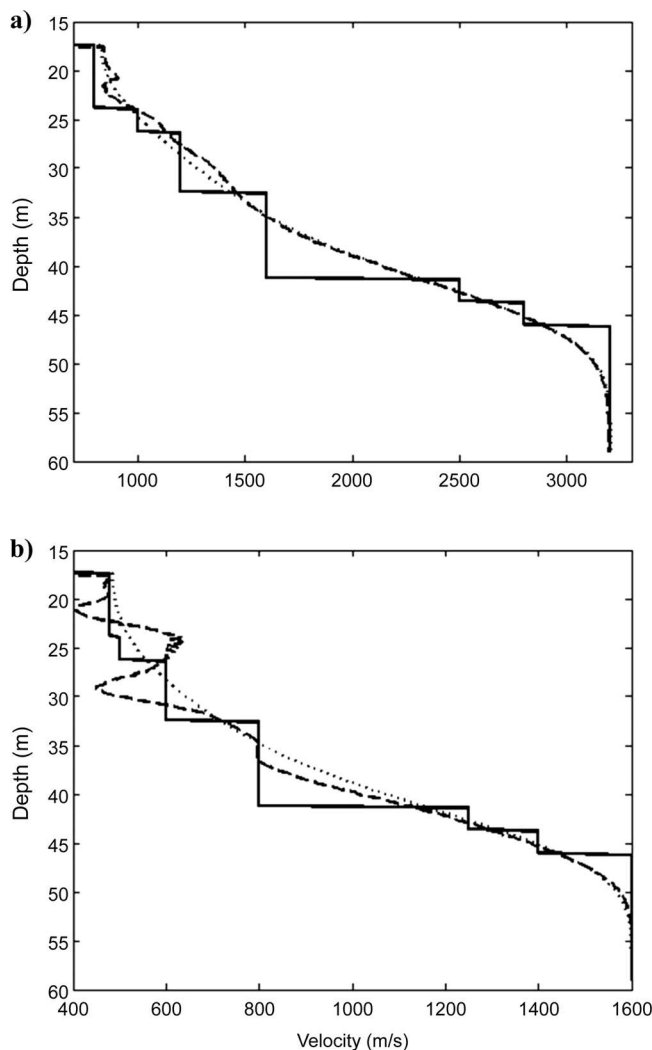


Figure 7. (a)  $V_P$  and (b)  $V_S$  parameter profiles extracted along a vertical line (located at a distance of 100 m) obtained after sequential inversion of seven frequencies varying from 21.3 to 134.3 Hz as indicated in Table 2. Profiles of the true model are plotted with solid black lines, the initial model is the dotted lines, and the final model is the dashed lines.



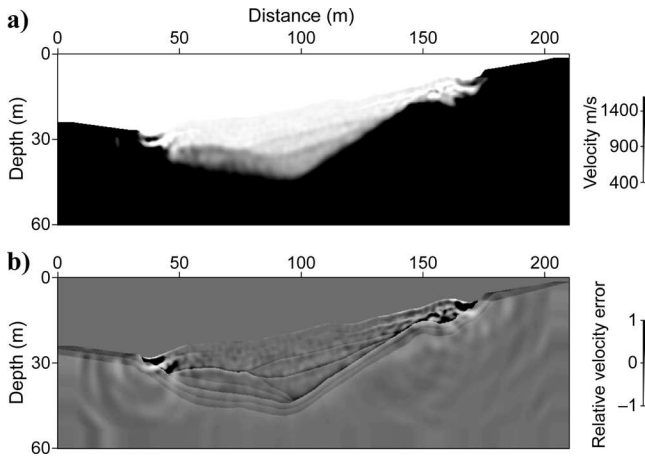


Figure 8. (a) Final  $V_S$  model and (b) relative velocity error (ratio of the velocity error to the true velocity), obtained after simultaneous inversion of three damped-frequency groups, varying from 21.3 to 134.3 Hz, as indicated in Table 2. Vertical and horizontal components are inverted. Significant velocity errors can be observed near the free surface.

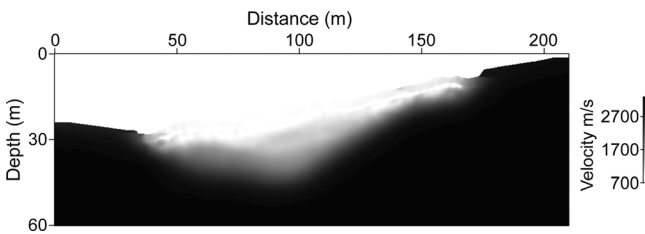
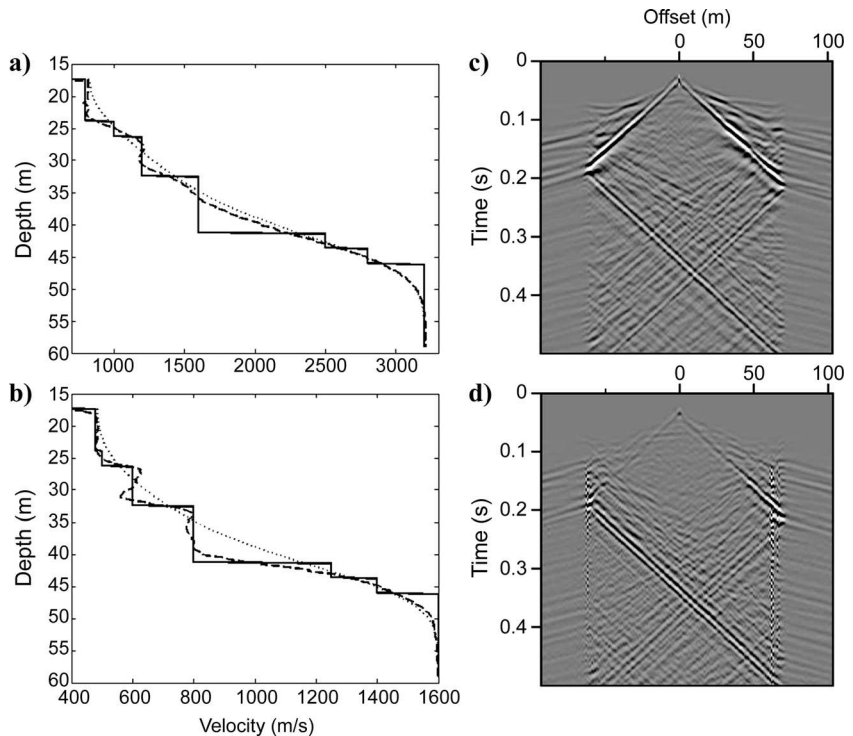


Figure 9. Final  $V_P$  model obtained after simultaneous inversion of three damped-frequency groups varying from 21.3 to 134.3 Hz as indicated in Table 2. Vertical and horizontal components are inverted.

Figure 10. (a)  $V_P$  and (b)  $V_S$  parameter profiles extracted along a vertical line (located at a distance of 100 m) obtained after simultaneous inversion of three damped-frequency groups, varying from 21.3 to 134.3 Hz, as indicated in Table 2. True model is plotted in solid black lines, initial model in dotted lines, and final model in dashed lines. (c) Vertical component of an initial differential seismic section (difference between data calculated with the true model and data calculated with the starting model). (d) Vertical component of a final differential seismogram (difference between data calculated in the final model and data calculated with the true model).



Amplitudes of the structures are significantly better defined for the shallow and the deep layers of the model in comparison with those obtained with the PCG optimization method (see Figures 8a, 9, 10a, and 10b). Fewer artifacts can be observed even in the shallow zone corresponding to the highest velocity contrasts.

Figure 12c depicts the evolution of the logarithm of the misfit function with respect to the iteration number with the L-BFGS and PCG methods for the first frequency group. The convergence level is drastically improved with the L-BFGS algorithm when compared to PCG. The final differential seismograms (Figure 14) show that amplitude residuals are strongly attenuated, even for the longer recording times. The strong residuals (observed in Figure 10d) from the highest velocity contrasts are significantly lessened. Similar effects have been noticed by Brossier et al. (2009) and attributed to the contribution of the off-diagonal terms of the Hessian matrix estimated by the L-BFGS algorithm. This confirms the promising prospects for realistic applications.

### Impact of data decimation

In this section, we analyze the impact of data decimation on the inversion results. The percentage of model degradation with respect to a reference case is estimated with the root mean square (rms) of the relative velocity error.

### Component selection

In practice, seismic investigations for shallow-structure characterizations are usually restricted to recording the vertical particle-velocity component. This limitation represents an additional ambiguity for the inverse problem.

Two configurations were tested by considering the vertical or horizontal component of the synthetic data. The same frequency

537 groups and damping terms were used as in the previous section  
 538 for consistency. The final distribution of  $V_S$  reconstructed by  
 539 considering the vertical component has a higher resolution (Fig-  
 540 ures 15a and 15c) compared to the one obtained by considering

the horizontal component. In the better case, we observe large 541  
 velocity errors close to the free surface in the zones with a very 542  
 high contrast between the steeply dipping bedrock and the shal- 543  
 lowest layer (Figures 15b and 15d). Percentages for  $V_S$  model 544  
 degradation (with respect to the reference case of Figure 8b) are 545  
 24% with the vertical component and 102% with the horizontal 546  
 component. As a result, the algorithm appears more sensitive to 547  
 the information provided by the vertical data component. 548

In addition, final inverted  $V_S$  images obtained by considering 549  
 the vertical component do not differ significantly from the ones 550  
 calculated by considering vertical and horizontal components. 551  
 However, a comparison of the figures of associated relative ve- 552  
 locity errors (Figures 8b and 15b) shows the lower resolution of 553  
 the deeper layers of the model. 554

Acquisition configuration 555

We finally investigate the sensitivity of inversion results to 556  
 the parameters of the recording geometry. The impact of 557

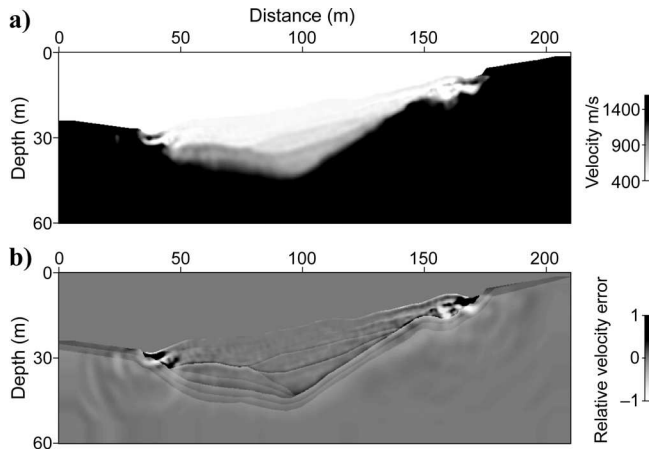


Figure 11. (a) Final  $V_S$  model and (b) relative velocity error (ratio of the velocity error to the true velocity), obtained after simultaneous inversion of three damped-frequency groups varying from 21.3 to 134.3 Hz. Five frequencies per group are used in the inversion. Horizontal and vertical components are considered.

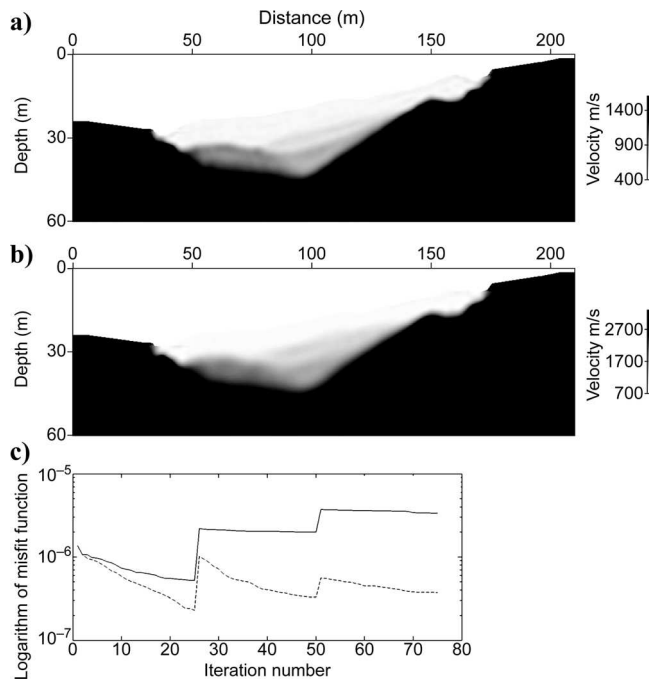


Figure 12. Final (a)  $V_S$  and (b)  $V_P$  models obtained after simultaneous inversion of three damped-frequency groups varying from 21.3 to 134.3 Hz with the L-BFGS algorithm. Three frequencies per group are used in the inversion. Horizontal and vertical components are considered. (c) Evolution of the L-BFGS and the PCG logarithm of the objective function with respect to the iteration number for the inversion of the first frequency group with three damping coefficients (see Table 2). Twenty-five iterations are performed for each damping coefficients. The L-BFGS algorithm drastically improves the convergence level of the objective function.

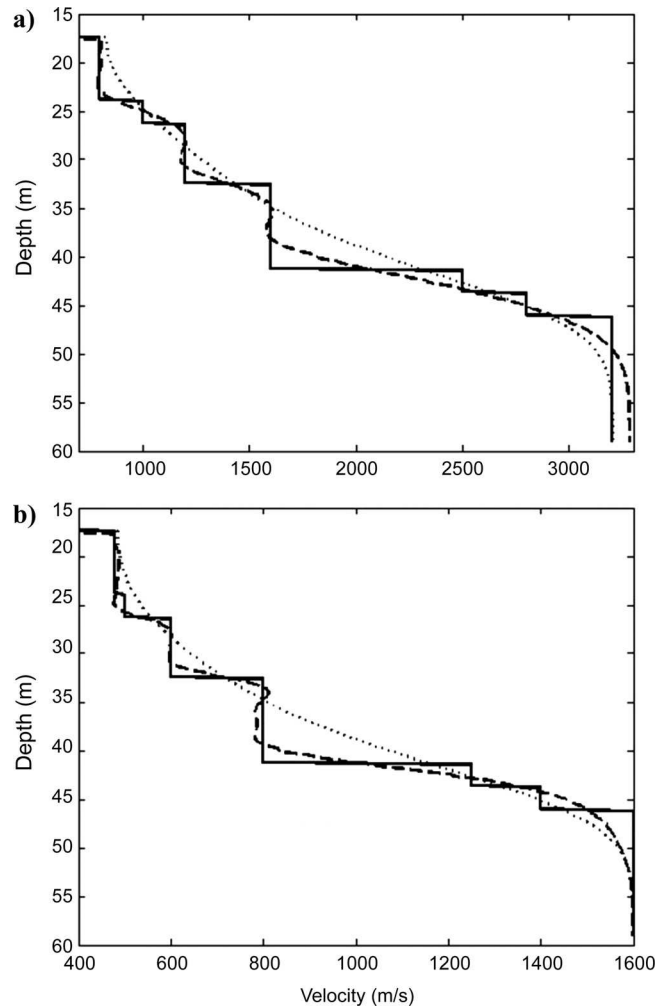


Figure 13. (a)  $V_P$  and (b)  $V_S$  parameter profiles extracted along a vertical line (located at a distance of 100 m) obtained after simultaneous inversion of three damped-frequency groups varying from 21.3 to 134.3 Hz as indicated in Table 2. The true model is plotted in solid black lines, the initial model is a dotted line, and the final model is a dashed line.

558 decimating survey geometries on the waveform tomography for  
 559 lithospheric imaging is addressed by [Brenders and Pratt \(2007\)](#).  
 560 The requirement of fully unaliased surface sampling  $\Delta_{\text{samp}}$  is  
 561 given by the relationship  $(\Delta r, \Delta s) \leq \Delta_{\text{samp}} = \lambda/2$ , where  $\Delta r$  and  
 562  $\Delta s$  denote receiver and source spacings and  $\lambda$  is the calculated  
 563 wavelength for a specific frequency with respect to the mini-  
 564 mum velocity of the medium. [Brenders and Pratt \(2007\)](#) suggest  
 565 that for a receiver spacing below  $\lambda/2$ , the image quality remains  
 566 acceptable for  $\Delta s \approx 3\Delta_{\text{samp}}$ . In the context of near-surface imag-  
 567 ing, field and logistic limitations often prevent the use of a

dense sampling of source array. We keep the number of receiver 568  
 constant and evaluate the effect of decimating the number of 569  
 sources with a sparser grid consisting of 99 sources (with a 2-m 570  
 spacing) and then 49 sources (with a 4-m spacing), respectively. 571  
 Recalling that the minimum wavelength with respect to the S- 572  
 wave velocity is computed from  $V_{S_{\text{min}}} = 480$  m/s and 573  
 $f_{\text{max}} = 134.2$  Hz, we have  $\Delta r = 1\text{m} \leq \Delta_{\text{samp}} = 1.78$  m and 574  
 $\Delta s < 3\Delta_{\text{samp}} \approx 3\lambda_{\text{min}}/2 \approx 5.34$  m for both cases. The minimum 575  
 and maximum offset coverages along the model are retained. 576  
 Only the vertical data component is considered for the 577  
 inversion. 578

Although the acquisition aperture is not modified, this acqui- 579  
 sition geometry is expected to mitigate the inversion perform- 580  
 ance slightly. Results show that a source sampling of 2 m leads 581  
 to acceptable results (Figures 16a and 16b) in comparison to 582  
 those obtained with a 1-m source sampling. The percentage of 583  
 model degradation is 12%. However, associated vertical profiles 584  
 depicted in Figures 17a and 17b demonstrate that the deep struc- 585  
 ture (with a  $V_S$  velocity of 800 m/s) is defined with a lower defini- 586  
 tion. A source sampling of 4 m introduces significant aliasing 587  
 effects near the free surface (Figures 16c and 16d), translated 588  
 into strong artifacts. The inversion obviously fails to converge 589  
 toward the true model with an acceptable level of resolution. 590  
 The final  $V_S$  image obtained is severely altered, with respect to 591  
 Figure 15a. The percentage of model degradation is 43%. 592

Extracted vertical profiles (Figures 17c and 17d) confirm the 593  
 aliasing effect observed near the free surface and the poor reso- 594  
 lution of the deep structures. This effect can be even more pro- 595  
 nounced in the presence of noise in real data, which seriously 596  
 mitigates the effectiveness of the algorithm to improve layer 597  
 definition. 598

**Computing time** 599

We used a constant mixed P0-P1 mesh with a total of 600  
 266,709 cells composed of 8232 P1 cells (with three degrees 601  
 of freedom per field) and 258,477 P0 cells (with one degree of 602  
 freedom per field), giving 1,415,865 degrees of freedom. The 603  
 forward modeling required an average time of 89 s to be solved 604

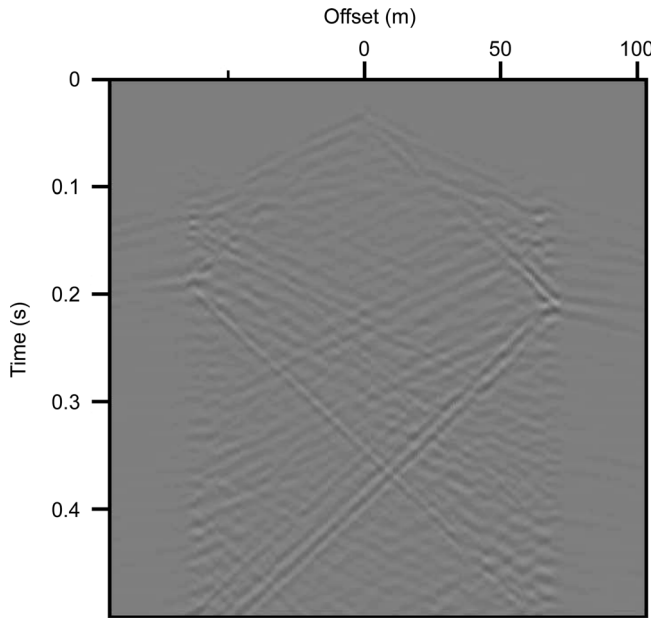


Figure 14. Vertical component of a final differential seismic section (difference between data calculated in the final model and data calculated in the true model) obtained after running the inversion with the L-BFGS algorithm. Amplitude residuals are strongly attenuated, compared with those obtained with the PCG algorithm (Figure 10d).

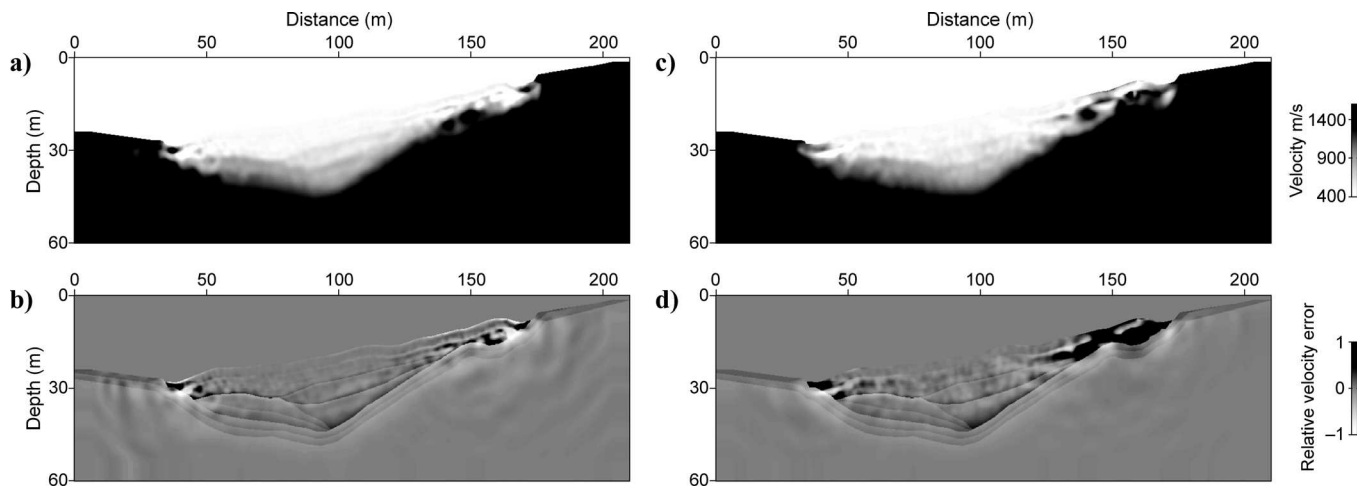


Figure 15. (a) Final  $V_S$  models and (b) relative velocity errors (ratio of velocity error to true velocity), obtained after simultaneous inversion of three damped-frequency groups varying from 21.3 to 134.3 Hz, as indicated in Table 2. Only the vertical (a, b) or horizontal (c, d) component data are inverted.

605 for 197 sources per modeled frequency. For the inversion, each  
 606 iteration required an average time of 400 s and a total memory  
 607 of 6.7 Gb for factorization. All simulations were performed  
 608 using 24 cores on a BRGM HP DL 165 G2 cluster, which consi-  
 609 sists of 32 nodes with Myrinet interconnection. Each node com-  
 610 prises two quad-core 2.3-GHz AMD Opteron processors, provid-  
 611 ing 16 Gb of RAM.

DISCUSSION

612 We have applied elastic FWI inversion to a realistic landslide  
 613 model characterized by strong lateral velocity variations and a  
 614 complex surface topography. In this particular context, the seis-  
 615 mic signal is dominated by surface waves that cannot be sepa-  
 616 rated easily from body waves because of the limited aperture of  
 617

the acquisition geometry. The inversion of raw data failed to  
 618 yield acceptable velocity images. This failure can be attributed  
 619 to the dominant contribution of surface waves. The effects of  
 620 surface waves on FWI have been investigated in small-scale  
 621 field experiments by Gélis et al. (2007) for the elastic case and  
 622 by Bleibinhaus and Rondenay (2009) in the presence of a com-  
 623 plex topography for the acoustic case on a larger scale.  
 624

The performed tests reveal that a combination of inversion of  
 625 overlapping groups of multiple frequencies and data damping to  
 626 allow a progressive introduction of the complex seismic events  
 627 (surface waves, multiples) is critical to mitigate the strong nonli-  
 628 nearities introduced by surface waves and to reconstruct the  
 629 shallow structures accurately. The use of a quasi-Newton L-  
 630 BFGS optimization algorithm can significantly improve the con-  
 631 vergence level of the method and the parameters reconstruction.  
 632

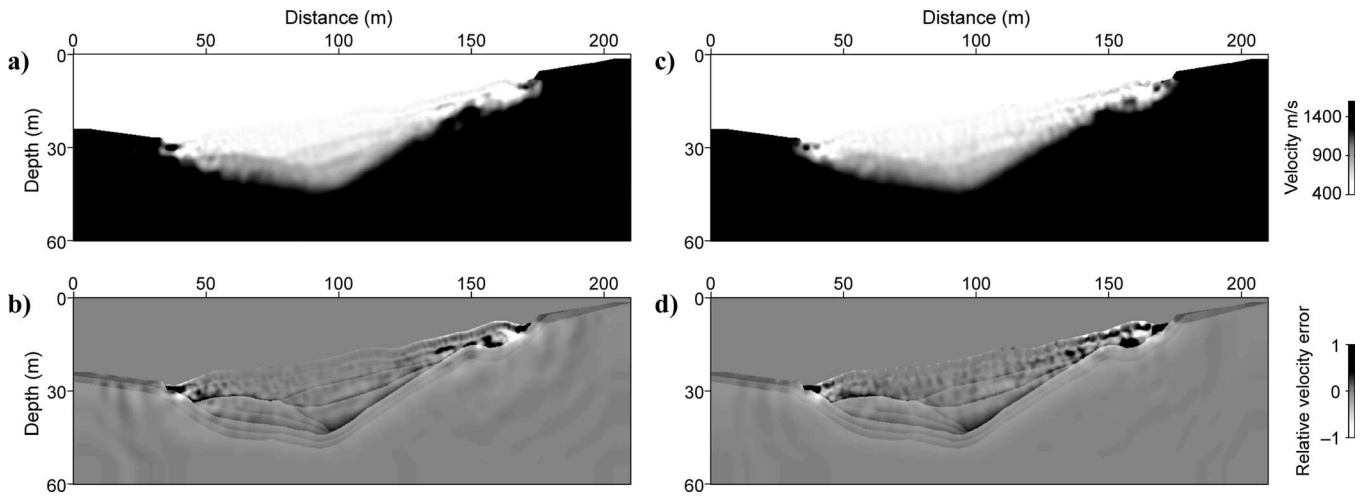


Figure 16. (a) Final  $V_S$  model and (b) relative velocity error (ratio of velocity error to true velocity), obtained after simultaneous inversion of three damped-frequency groups varying from 21.3 to 134.3 Hz as indicated in Table 2. Only the vertical component is inverted and a decimated acquisition ( $\Delta s = 2$  m) is used. (c, d) Final  $V_S$  model and relative velocity error with  $\Delta s = 4$  m.

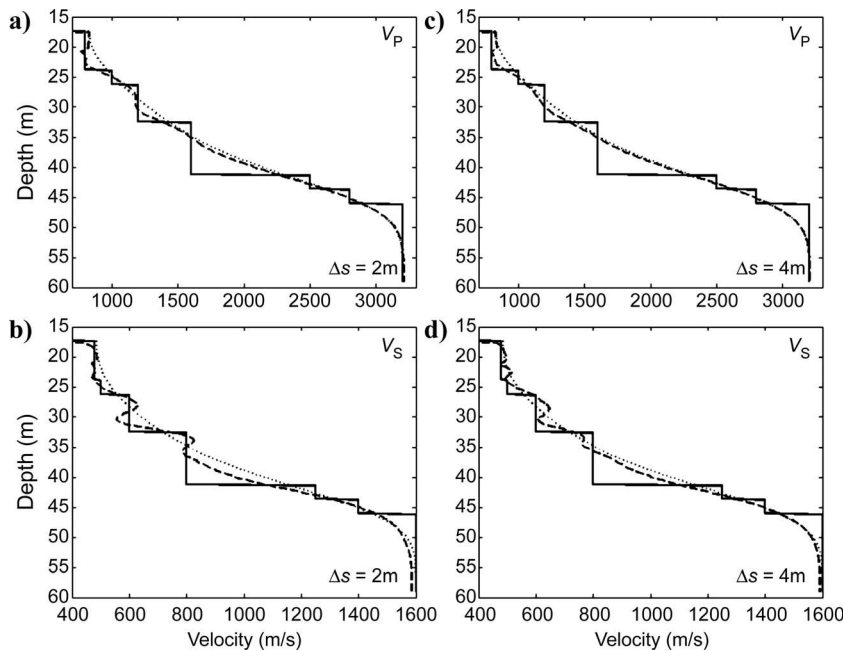


Figure 17. (a, c)  $V_P$  and (b, d)  $V_S$  parameter profiles extracted along a vertical line (located at a distance of 100 m), obtained after simultaneous inversion of three damped-frequency groups varying from 21.3 to 134.3 Hz, as indicated in Table 2. Only the vertical component is inverted. Two levels of decimation are considered with a source sampling of 2 m (a, b) and 4 m (c, d) along the surface topography. True model is plotted in solid black lines, initial model is plotted in dotted lines, and final model is plotted in dashed lines.

633 Efficiency of the process can, however, be severely altered by  
634 an insufficient source sampling interval.

635 An important extension to the elastic FWI, which may be crit-  
636 ical for challenging real data applications, should incorporate  
637 the reconstructed attenuation parameters  $Q_P$  and  $Q_S$ . Implemen-  
638 tation of the algorithm in the frequency domain can take advant-  
639 age of the complex-velocity Kolsky-Futterman model (Toverud  
640 and Ursin, 2005). Application to real data, in the context of  
641 near-surface characterization, can be based on the waveform-to-  
642 mography workflow proposed by Smithyman et al. (2009) to  
643 produce images of  $V_P$  and  $Q_P$  parameters and to locate shallow  
644 targets. It also requires an estimation of the source (Pratt, 1999),  
645 which was supposed known in our work.

## 646 CONCLUSION

647 We have presented a numerical study to evaluate the potential  
648 of a 2D FWI approach; it shows promising prospects for imag-  
649 ing shallow structures in the presence of a complex topography.  
650 A discontinuous Galerkin method, based on a low-order mixed  
651 P0-P1 interpolation, is used for accurate wavefield modeling at  
652 a reasonable computing cost. A 2D elastic frequency-domain  
653 FWI algorithm has been applied to a realistic landslide model,  
654 characterized by highly contrasted layers and strong lateral ve-  
655 locity variations.

656 A two-level preconditioning strategy, based on simultaneous  
657 multifrequency inversion of damped data, has been applied to  
658 mitigate difficulties inherent in classical single-frequency inver-  
659 sions. Results confirm that simultaneous inversion of damped  
660 data, which allows a progressive introduction of converted and  
661 free surface waves, significantly outperforms the successive single-  
662 frequency inversion approach. It is a useful solution to miti-  
663 gate strong nonlinearities resulting from surface waves and to  
664 avoid convergence toward a local minimum. We have also  
665 emphasized the high potential of the L-BFGS optimization  
666 method to improve the convergence level significantly, com-  
667 pared to the more classical PCG algorithm.

668 Finally, we have addressed the impact of some recording pa-  
669 rameters on medium reconstruction. We have shown that  
670 restricting the inversion to the vertical component data can lead  
671 to acceptable results in terms of imaging resolution and conver-  
672 gence level, with a percentage of model degradation of 24%.  
673 We have also illustrated how poor model illumination is trans-  
674 lated in terms of imaging resolution. Future work will tackle the  
675 construction of the initial model, a key issue for FWI before  
676 considering applications to real data.

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