A Generalized Approach for Modeling 3D Transient Free and Moving Boundaries in Coastal Aquifers

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ABSTRACT

Coastal aquifers involve varying conditions in time and space, owing to the occurrence of free and moving boundaries, such as the water table, seepage face and saltwater intrusion interface. A fast Updating Procedure (FUP) is developed for solving these interfaces in steady and transient conditions with the finite element method. Several test examples, which involve confined and phreatic groundwater flow in coastal aquifers, are studied to demonstrate the FUP capability of predicting accurate results. Comparisons of the FUP numerical results are made against available analytical solutions, laboratory experiments measurements and other computer codes. To show also the capability of this technique to solve real field situations, it was applied to the coastal aquifer of Martil in Morocco to study and understand the aquifer response to changes in recharge and total rate of pumping water, and their effects on seawater intrusion.

INTRODUCTION

The problem of saltwater intrusion can be treated with two methods, one considers that saltwater and freshwater are miscible and that a transition zone exists [Voss, 1984], in which situation miscible density dependent groundwater flow models are traditionally used. The other method is based on an abrupt approximation [Bear, 1972; Huyakorn et al., 1996; Larabi and De Smedt, 1997; Sbai, 1999] and for which (i) the transition zone is thin relative to the thickness of the freshwater lens, (ii) freshwater and saltwater are immiscible, and (iii) the freshwater and saltwater are separated by a sharp interface. Additionally, the approach considering flow in both fresh and salt water regions, leads to a coupled system of equations to be solved. This is cumbersome for a fully three dimensional model owing to numerical constraints and higher computational costs involved. Instead, a new numerical procedure is developed, assuming a sharp interface between freshwater and saltwater, for which the location, shape and extent must be determined.

A first steady state version of this model was presented by [Larabi and De Smedt, 1997], who elaborate a part of the numerical technique which will be described, and also validate the developed model by running several benchmarks. In this work an enhanced version of their steady state model is developed, several features were implemented as a robust solver based on an M-matrix preconditioned incomplete factorization. Also, special boundary conditions such as the seepage face, and sea outflow face are efficiently handled. But, the most important achievement is the development of a transient model for simulating the moving sharp interface and the moving water table in case of free surface flow conditions. Numerical algorithms used

as a basis for this model are similar to those for the steady state model for updating unsaturated hydraulic conductivity, with a small exception. The main difficulties due to the strong non-linearly of the equations systems, and which are related to estimation of the capacitance matrix coefficients or the so-called cord slope approximation, are tackled for the case of moving interfaces by adopting an idealized soil characteristic curve whose parameters are internally estimated, and which are depending on the local saturation in the interfaces neighborhood, and the finite element mesh configuration. This strategy proves to be very efficient in combination with a modified Picard iteration featuring a variable relaxation technique. A full description of the theory and the methodology used for developing GEO-SWIM code is found in [Sbai, 1999]. In this article the key developments are highlighted, followed by an exposition of theoretical problems for which the technique has been validated, and a realistic field application of the model to the Martil aquifer in the northern part of Morocco.

THEORETICAL DEVELOPMENTS

The nonlinear partial differential equation governing the variable saturated flow in 3D heterogeneous porous medium [Huyakorn and Pinder, 1983] is given by

$$S(h)\frac{\partial h}{\partial t} = \nabla(\mathbf{K}\nabla h) + R \tag{1}$$

Where $S(h) = S_s + d\theta/dh$ is the general storage term $[L^{-1}]$ the first term being the specific storage coefficient and the second being the water capacity, h is the groundwater potential [L], $\mathbf{K}$ is the unsaturated hydraulic conductivity tensor $[LT^{-1}]$ depending upon position and pressure or degree of saturation, R is the source/sink term $[T^{-1}]$, and ∇ is the del operator $[L^{-1}]$, $(\partial/\partial x, \partial/\partial y, \partial/\partial z)^T$, where $\mathbf{x} = (x,y,z)^T$ is the spatial vector [L], and t represents time [T]. In order to have a unique mathematical problem, boundary conditions and the initial potential distribution $h(\mathbf{x},t)$ are necessary. The location of the water table and the fresh-saltwater interface are unknown a-priori and hence the nonlinearity of the problem rises in consequence.

The finite element method in space using hexahedral elements, coupled within a fully implicit finite difference method in time, are used to solve (1). This results in a nonlinear system of algebraic finite element equations. in matrix form at the $(k+1)^{th}$ time step t_{k+1}, $k = 0, 1, \ldots$

$$\left([\mathbf{G}] + \frac{[\mathbf{S}]}{\Delta t}\right)\{\mathbf{h}\}^{t_{k+1}} = \{\mathbf{B}\} \tag{2}$$

Where $\{\mathbf{h}\}^{t_{k+1}}$ is the unknown vector containing the nodal potentials at time t_{k+1}, $[\mathbf{G}]$ is the global conductance matrix depending on the geometrical and conductive properties of the flow domain, $[\mathbf{S}]$ is the diagonalized storage matrix, and the right hand side vector $\{\mathbf{B}\}$ incorporates boundary conditions together with sources, sinks, and storage terms at time step t_k.

In the present approach a fixed finite element mesh is used to discretize the whole domain, including saturated, unsaturated and saltwater regions. The groundwater flow is assumed to occur only in the saturated freshwater zone, which means that the flow in the unsaturated and saltwater regions are negligible compared to the freshwater flow. The saltwater interface and the water table are effectively considered as impervious boundaries, but not as boundary conditions of the problem.

The fast updating procedure technique

This technique involves implicit updating of the saturated flow field independently of the unsaturated and saltwater zones, while they are still included in the meshed domain. This is performed in two steps, the first concerns the approximation of the relative hydraulic conductivity between nodal points in the neighborhood of the water table and the fresh-saltwater interface, this method is described in [Sbai et al., 1998; Sbai and De Smedt, 1998a] and in more detail in [Sbai, 1999].

The second part of the algorithm, approximates the nonlinear storage or the time dependent term in the right hand side of (2). The nodal storage depends on the water table and the saltwater interface positions, and is evaluated as

$$V_i \frac{d\theta}{dp} \tag{3}$$

Where V_i is the control volume patch centered on node i, and p is the water pressure [L]. The method used to evaluate the derivative term in (3) affects significantly the convergence behavior of the iterative schemes, due to steep gradients and discontinuties or points of inflection in the soil curves. The functionals showed in Fig. 1 are used to achieve the numerical differentiation method adopted in this model. At the beginning of each time step the differential in function (a) is directly used to approximate the chord slope, in the next iterations nodal water content values are relaxed following the function (b) and the differential expression is calculated thereafter. The limiting depth parameter d used for the water table zone is evaluated as

$$d_i = \frac{V_i}{S_i} \tag{4}$$

Where S_i is the projected surface area attributed to node i over the horizontal plane. The storage variations due to changes of the saltwater zone displacement are evaluated to be equal to the saturated water content θ_s. The newly distinguished pressure distribution around the fresh-saltwater interface is smaller in size in comparison with the equivalent pressure distribution existing around the water table, because small variations in the water table position involves greater displacement of the fresh-saltwater interface. And, hence much larger variations in the storage term. The coefficient of proportionality is taken equal to the density difference ratio δ.

The merit of the FUP procedure results from the fact that only relative hydraulic conductivity and nodal storage values have to be adapted, while the finite element

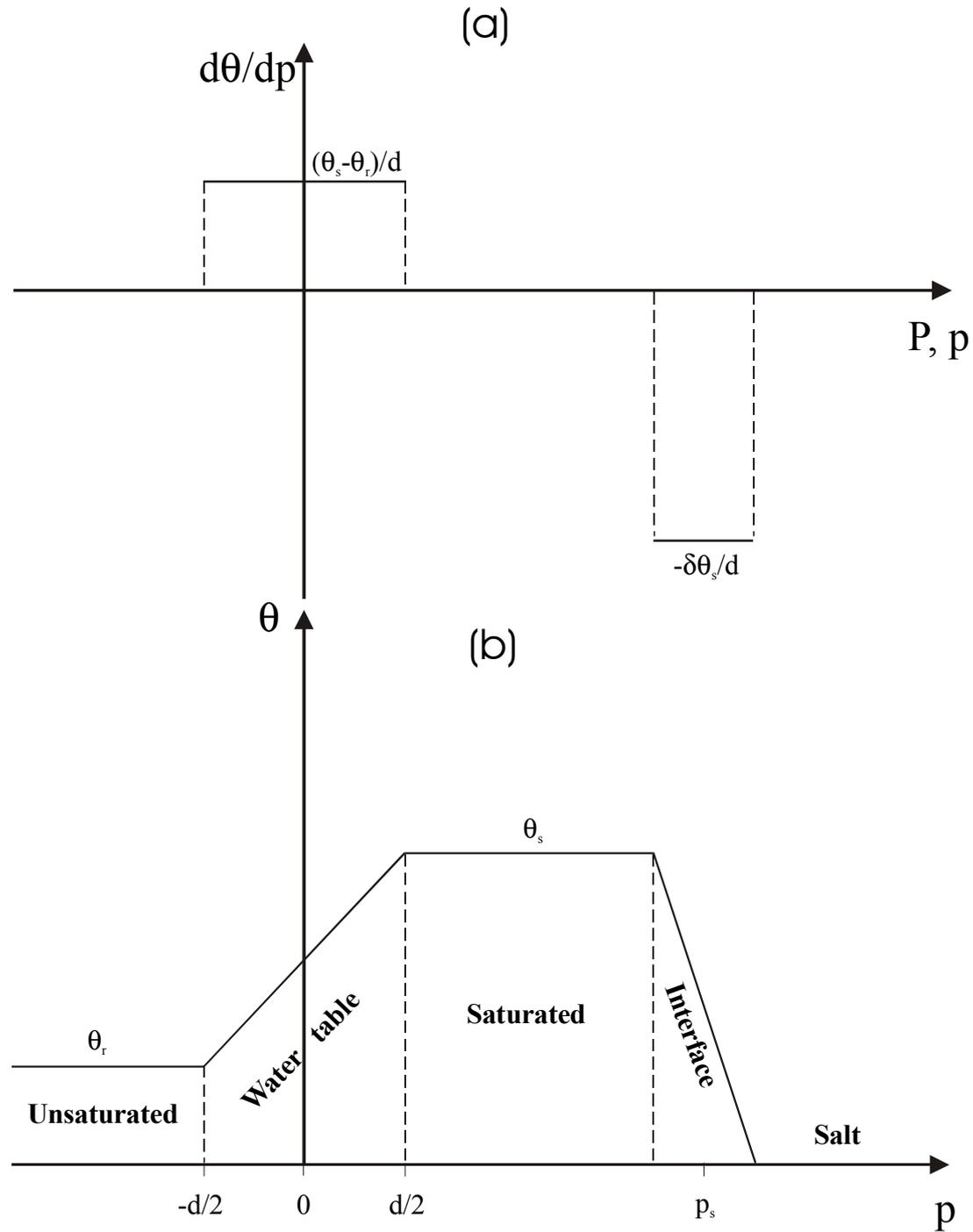mesh and saturated conductance coefficients remain fixed, such that the method is computationally fast.

Figure 1. Saturation curves used in the model (a) at time periods start up and (b) during the iterative process.

MODEL APPLICATIONS AND RESULTS

Free surface flow and seawater intrusion in coastal aquifers

Several test examples were studied in steady conditions analytical [Larabi and De Smedt, 1997] and time dependent conditions [Sbai et al., 1998] to demonstrate the

numerical technique capability of predicting accurate results efficiently. A comprehensive validation of the model predictions is achieved by comparing to laboratory experimental measurements of 3D moving free surface flow in earth dam model and a moving saltwater interface in a 3D laboratory sand box model of irregular shape [Sbai, 1999]. These problems are rather complex because they allow for real 3D flow and for heterogeneous dam material. The predicted 3D results compare well with the measurements. More other applications dealing with free surface flow and saltwater intrusion, have been also studied for comparison with other numerical schemes, but not included in this paper.
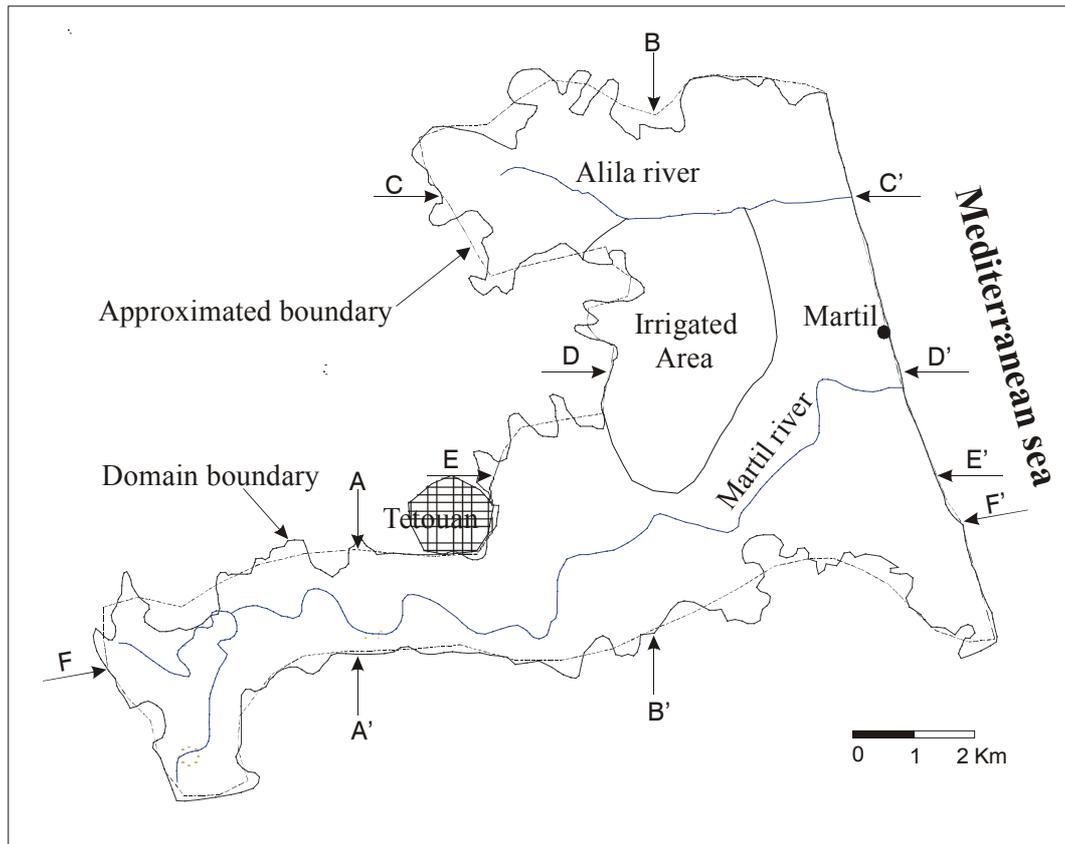

Figure 2. Study area and cross-sections locations.

The Martil Coastal aquifer (Morocco)

The Martil aquifer system is located in the northwestern part of Morocco on the Mediterranean coast (Fig. 2). It is the important local aquifer system in the region used for public supply, irrigation and industry in Tetouan city. In recent years, the aquifer has become vulnerable to potential pollution, especially due to seawater intrusion from the Mediterranean sea.
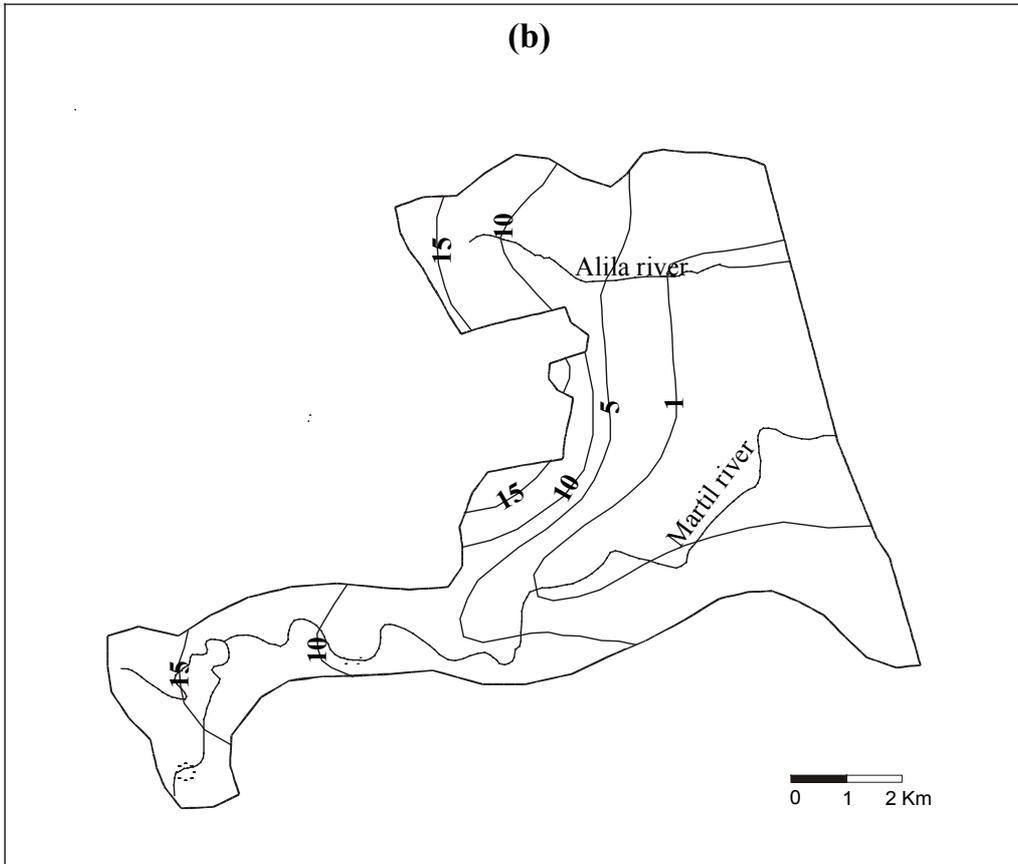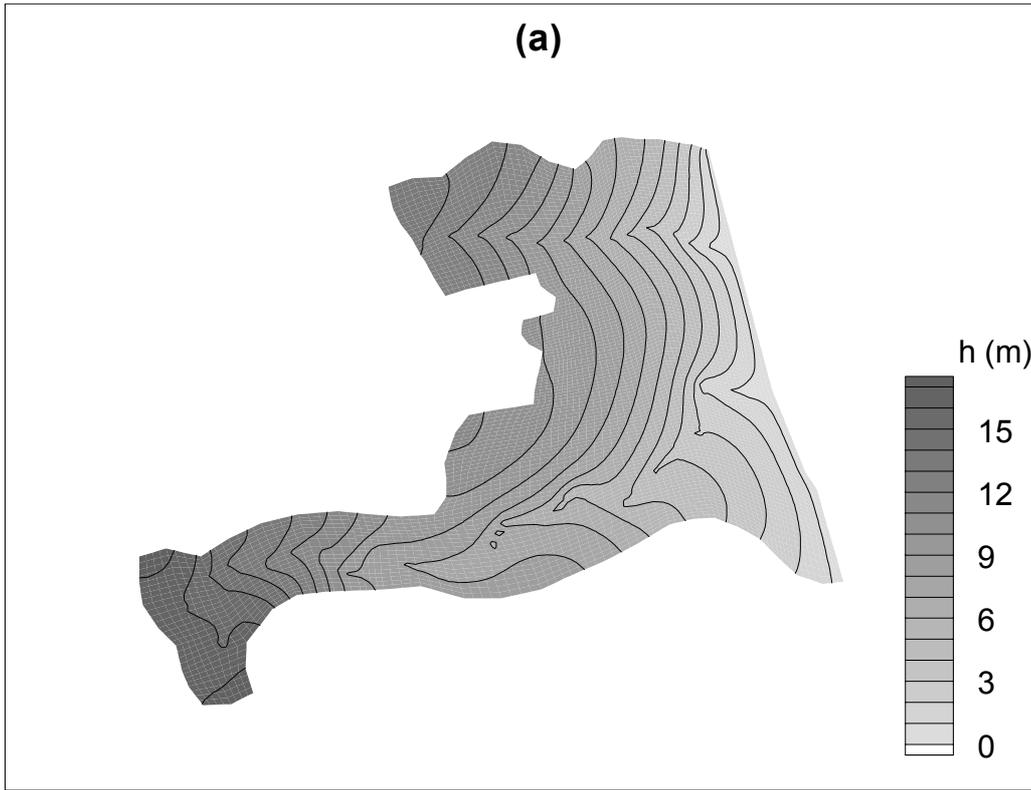
Figure 3. Comparison between (a) computed and (b) observed steady state groundwater potentials (presented as meters above sea level).

A three-dimensional finite element model of this aquifer is developed using the FUP procedure. First, a steady-state groundwater flow is simulated. Calibration of this model will establish natural conditions and hydraulic conductivity ranges for the different aquifers. Afterwards a transient simulation is performed to predict future lateral extension of the saltwater encroachment due to pumping of groundwater.

The aquifer system consists of two aquifer units, separated by a leaky aquitard. The upper aquifer is formed of Quaternary alluvial deposits of the Martil river; the lower aquifer unit is composed of sandstone-limestone Pliocene formations, while the aquitard is mainly marl and clay. Variations in thickness are shown to be significant from North to South, and also along the West-East direction. To better characterize the thickness of the three hydrogeological units a reinterpretation of data obtained from a total of 59 wells and boreholes is performed with support of *GIS* tools.

The conceptual model used in this study is a multi-layer aquifer system of variable thickness. This aquifer system is assumed isotropic and homogeneous with each sub-region. A structured surface mesh of 93 columns and 121 rows is used to approximate the aquifer domain, and some sub-zones are set to be inactive to fit the remaining part at the domain boundaries, especially at the eastern boundary near Tetouan city. In total, the aquifer system is divided into 9 nodal layers, with 88320 hexahedral elements and 101277 nodes. The finite element mesh used for the numerical simulation of the Martil aquifer system which is adjusted to fit the structure and the extension of the hydrogeological layers and the wells locations. Domain boundaries are set impervious, except at the eastern part, where the aquifer is in direct contact with the sea (outflow face boundary condition). The extent of the fresh water outflow to the sea is automatically determined by the model as part of the results. Nodes along the rivers paths are taken as fixed head with the assumption that water levels are equal to the elevation. Another condition taken into account is the effective recharge, which is assumed uniformly distributed over the whole surface basin. Saturated hydraulic conductivity values are deduced from calculated transmissivities of pumping tests analysis, conducted by the Regional Hydraulic Department of Tetouan.

A steady state simulation is performed under natural conditions (i.e. no pumping wells). The objective of this calibration process is to reproduce a natural groundwater flow pattern of the aquifer system and at the same time provide a range of confidence limits for the model parameters. The obtained results for each test run were compared to piezometric levels measured in 1966, for which it is assumed that the aquifer was not yet heavily pumped at that time. A trial and error calibration procedure is used to estimate the hydraulic conductivity of different layers to best reproduce the reference piezometric levels (1966). The tests show that the conceptual model is more sensitive to changes in the effective recharge value. Computed groundwater potential heads versus observed values are compared in Fig. 3, the fit is satisfactory except at the center of the plain and near the coastline where differences exist.

In general, the model is able to reproduce the same flow pattern, indicating that the main groundwater flow is directed W-E, with some convergence tendency to the rivers. the fresh-salt water interface is computed iteratively in parallel with the groundwater potential heads. W-E cross-sections displayed in Fig. 4 show the shape of the steady interface and water heads distribution in different parts of the aquifer. By comparing cross-sections C-C' and E-E' which are respectively parallel to the Alila and Martil rivers, it follows that saltwater intrusion is more sensitive along the Martil river as the hydraulic gradient is the smallest due to the flat topography. The parameters obtained after calibration are: saturated hydraulic conductivity K_s =

3.5m/d, and porosity n = 0.4 in the alluvial formation; saturated conductivity K_s = 0.05m/d, and porosity n = 0.25 in the clay/marl aquitard; saturated conductivity K_s = 4.5m/d, and porosity n = 0.25 in the Pliocene limestone-sandstone formation; a fresh-saltwater density difference ratio δ = 0.03; and an estimated effective recharge value of 50mm/y.
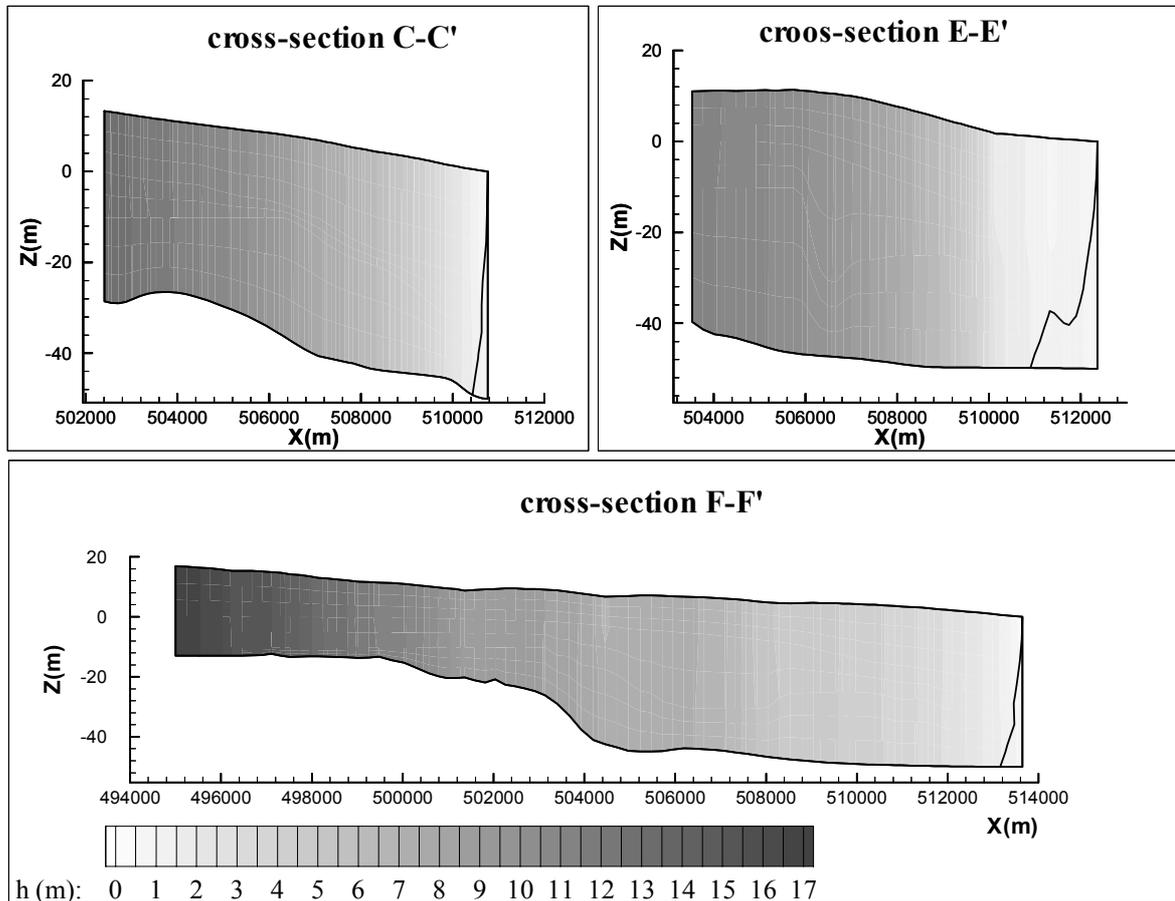

Figure 4. Cross sectional views of the computed groundwater heads and fresh-salt water interface position.

Future prediction scenario for seawater intrusion. A second set of simulations is performed, to predict the present and future groundwater flow and seawater intrusion in the aquifer. A long-term transient simulation is made of 40 years starting in 1966, and using the steady state conditions from the first set simulations as initial head values. The pumping of groundwater is assumed to decrease the natural recharge by half during this period. Fig. 5 shows the positions of the computed moving interface plotted along cross-section F-F'.
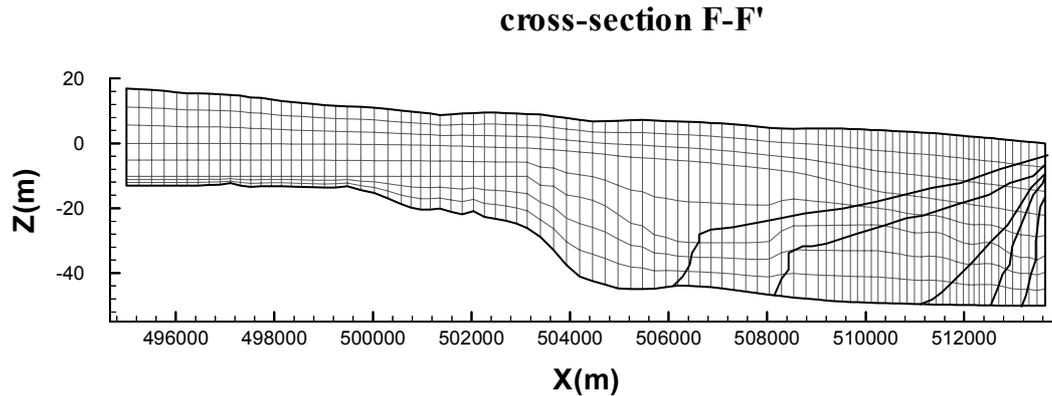
cross-section F-F'

Figure 5. Cross-sectional view of the seawater intrusion; simulated moving interface positions from 1974 to 2006 with 8 years interval are plotted.

CONCLUSION

A fast updating numerical procedure is developed for solving 3D groundwater flow involving saltwater intrusion and phreatic flow in steady and transient conditions. The model uses a fixed FE mesh, and iteratively adjusts the water table and the saltwater interface position by neglecting the flow in the unsaturated and the saltwater zone. This is achieved without fully reconstructing the conductance and capacitance coefficients matrix of the FE element equations. During the solution procedure, the saturated conductance and capacitance coefficients are constant. For the conductance matrix, only the relative conductivities have to be adjusted during each iterates, depending on the status of water at each node. The capacitance matrix coefficients are also adjusted in the unsaturated and saltwater zones depending upon the change in the storage. The proposed numerical approach has been verified using a series of test problems under steady and transient flow situations. The comparison study has shown that the technique is accurate when compared with available analytical solutions. Validation of the model is also made with respect to measurements from a 3D laboratory sand box model and other numerical schemes.

This model is applied to the aquifer system of Martil situated in Morocco using the FUP numerical procedure, in order to study seawater intrusion effects in terms of the shape and lateral extension. This study enables to understand the aquifer response to changes in recharge and total rate of pumped water, and their effects on seawater intrusion. Different scenarios are investigated for the period of 1966 to 2006, to predict future situations and the salinization risk from seawater intrusion. The obtained results show that the interface will move fast and travel over considerable distances in forthcoming years, and will produce an irreversible degradation of the groundwater quality. An alarming optimal management scheme to adopt in the near future is necessary for its safeguard. Sine this is the first time that a simulation model for groundwater flow and seawater intrusion in the Martil aquifer system is performed, this model should be improved as further data are obtained from the field.

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